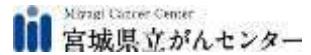


# 自己紹介



# 自己紹介



地理・当院のある宮城県名取市は…



仙台空港



北限しらす



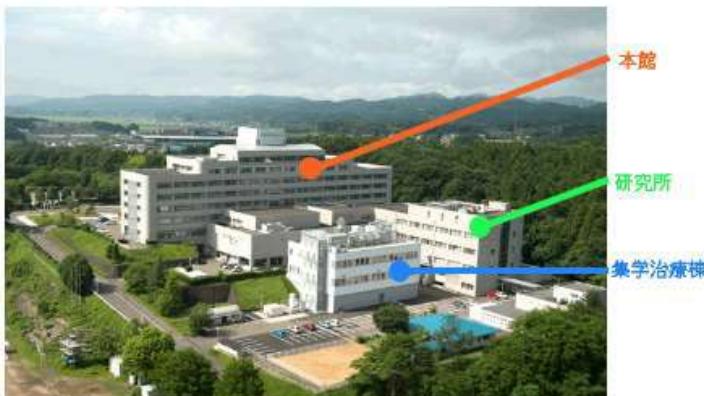
閑上の赤貝



仙台せり鍋

# 自己紹介

## 病院背景・概要



### 東北唯一のがんセンター

宮城県立がんセンターは昭和42年に開設された  
宮城県成人病センターが前身であり、平成5年に現在の宮城県立がんセンター  
となっています。

### 都道府県がん診療拠点病院

### がんゲノム医療連携病院

診療科 26科

病床数 383床

平均外来数 330人

平均在院日数 15日

検査件数 1日4700検査 (生化学・免疫系2950検査)

検体検査管理加算IV

職員数 504名 (臨床検査技師 24名)

### **ISO 15189 取得**

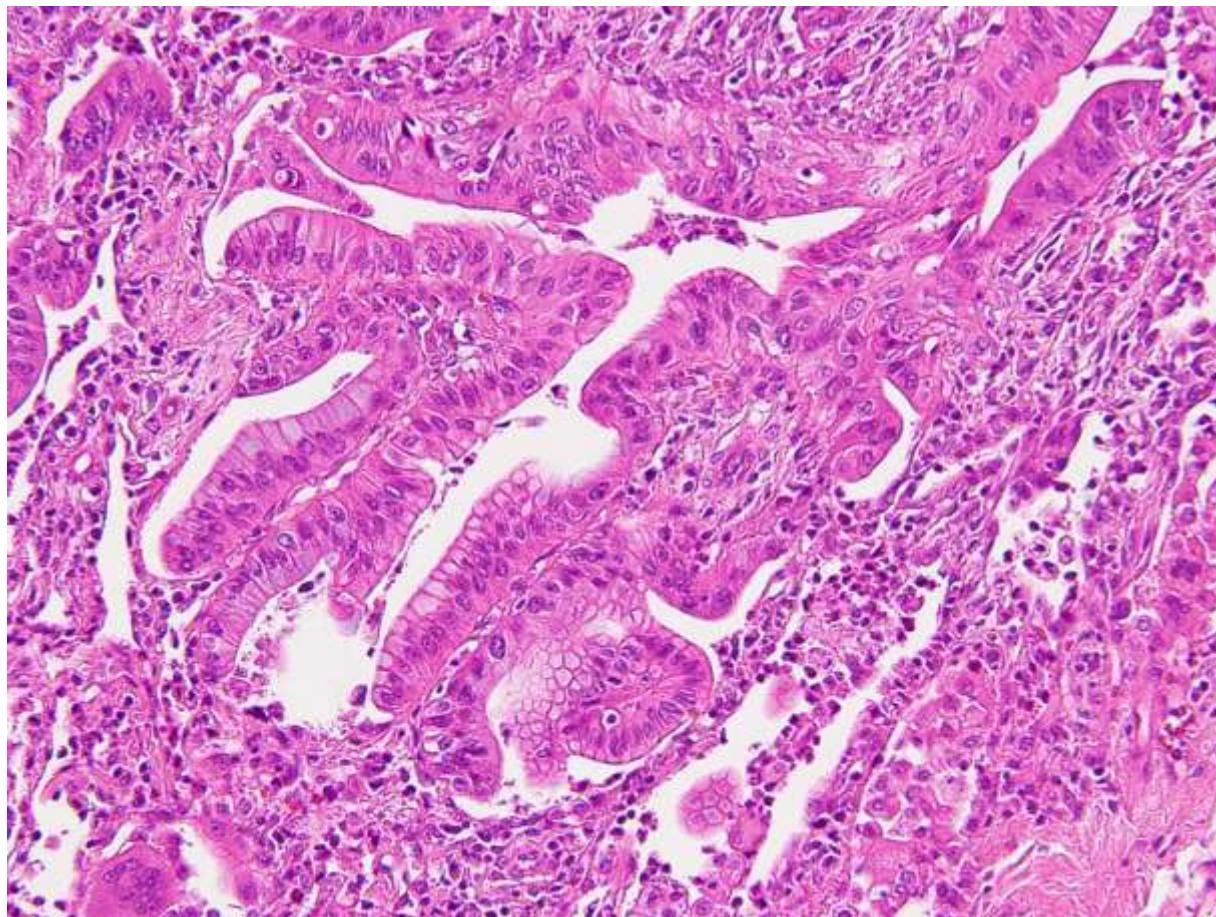
# 富士川鉄橋と新幹線



# 宮城県立がんセンターと新幹線



# 肺腺癌の顕微鏡写真



# Microscopic examination

- Shape and form
- Arrangement
- Structure



**report**



# 仙台発祥



# Fibonacci numbers, binary quadratic forms and topographs

- Univ. of Tokyo
- 9/Nov, 2025
- Ikuro SATO
- Miyagi Cancer Center

# □を埋めよ

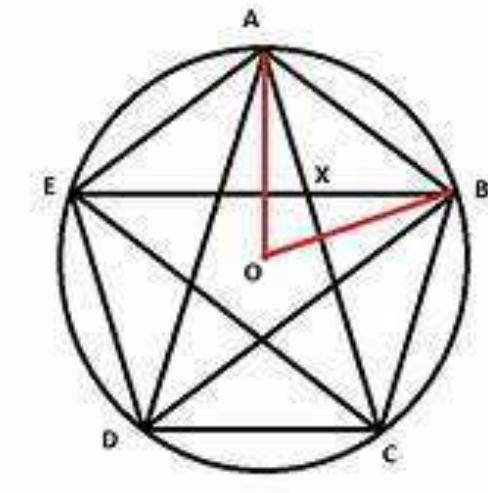
- 1,3,5,7,9,11,13,□,,,
- 1,2,4,8,16,32,64,□,,,
- 1,1,2,3,5,8,13,21,□,,,

# Fibonacci numbers

Fibonacci numbers: $F[n+1] = F[n] + F[n-1]$ , $F[1] = 1$ , $F[2] = 1$										
$F[n]$	1	1	2	3	5	8	13	21	34	55
$F[2n-1]$	1		2		5		13		34	
$F[2n]$		1		3		8		21		55
$F[n]$	89	144	233	377	610	987	1597	2584	4181	6765
$F[2n-1]$	89		233		610		1597		4181	
$F[2n]$		144		377		987		2584		6765

# Fibonacci numbers

- 1,3,5,7,9,11,□,,,
- 1,2,4,8,16,32,□,,, exponential,  $2^n$
- 1,1,2,3,5,8,13,□,,, quasi-exponential,  $\sim \phi^n$
- Golden ratio:  $\phi = (1 + \sqrt{5})/2 = 1.618$
- $\phi^{-1} = \phi - 1 = (-1 + \sqrt{5})/2 = 0.618$
- $\phi^2 = \phi + 1 = (3 + \sqrt{5})/2 = 2.618$



# Fibonacci numbers

- $\{F_n\} = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \dots\}$
- $f(p/q) = (p+q)/p$  (Diophantine approximation)
- $p_{n+1} = p_n + q_n, q_{n+1} = p_n$
- $1/1, 2/1, 3/2, 5/3, 8/5, 13/8, 21/13, \dots \rightarrow \phi = (1 + \sqrt{5})/2$

# Fibonacci numbers

- $\{F_n\} = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \dots\}$
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- Denominator of best approximating fraction

# Fibonacci numbers

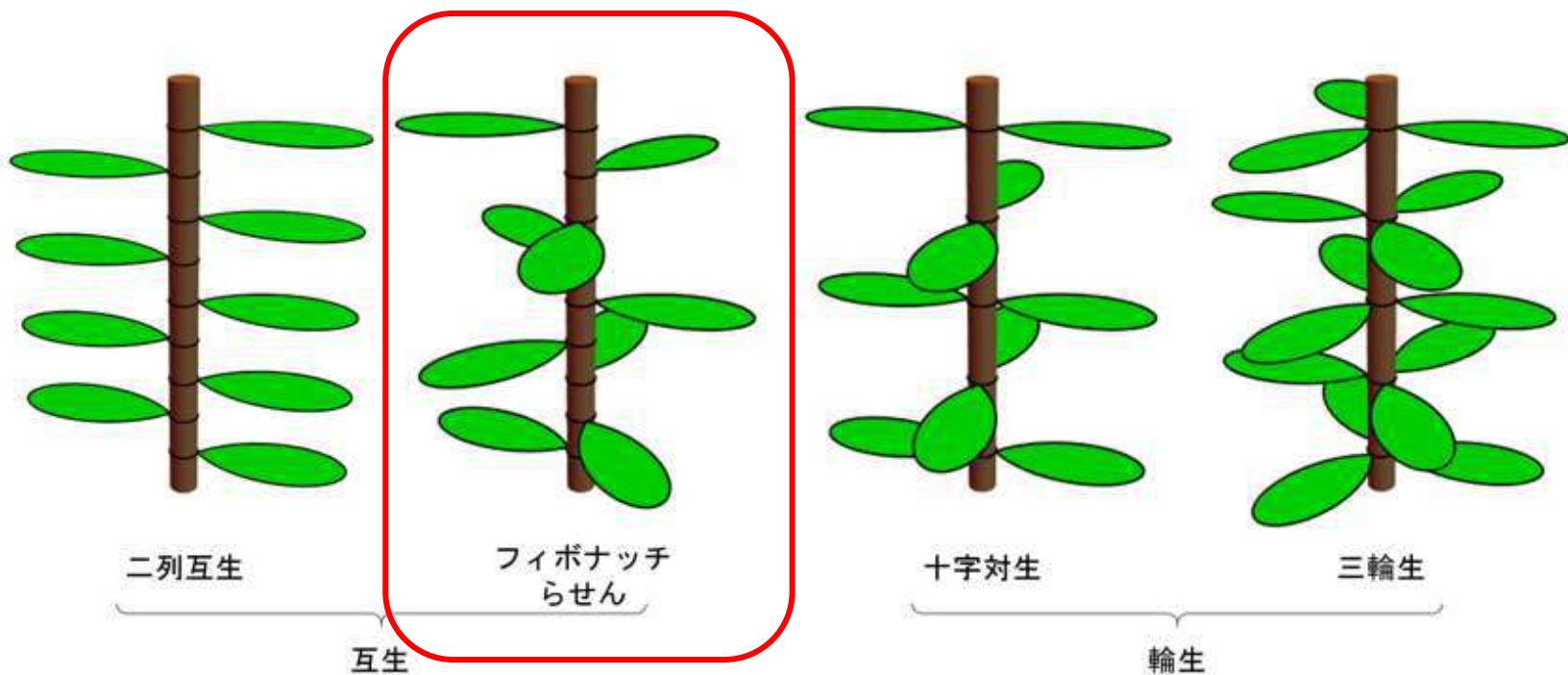
- $\{F_n\} = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \dots\}$
- $f(p/q) = (p+q)/p$  (Diophantine approximation)
- $p_{n+1} = p_n + q_n, q_{n+1} = p_n$
- $1/1, 2/1, 3/2, 5/3, 8/5, 13/8, 21/13, \dots \rightarrow \phi = (1 + \sqrt{5})/2$
- Denominator of **best approximating fraction**
- $p^2 - (p+q)q = p^2 - pq - q^2 = (-1)^n, p = F_{n+1}, q = F_n$
- $p^2 - pq - q^2 \neq 0$ , if  $(2p-q)^2 - 5q^2 = 0$  then  $\sqrt{5}$  is rational.
- $1/1 < 3/2 < 8/5 < 21/13 <<< \phi <<< 13/8 < 5/3 < 2/1$

# 植物編

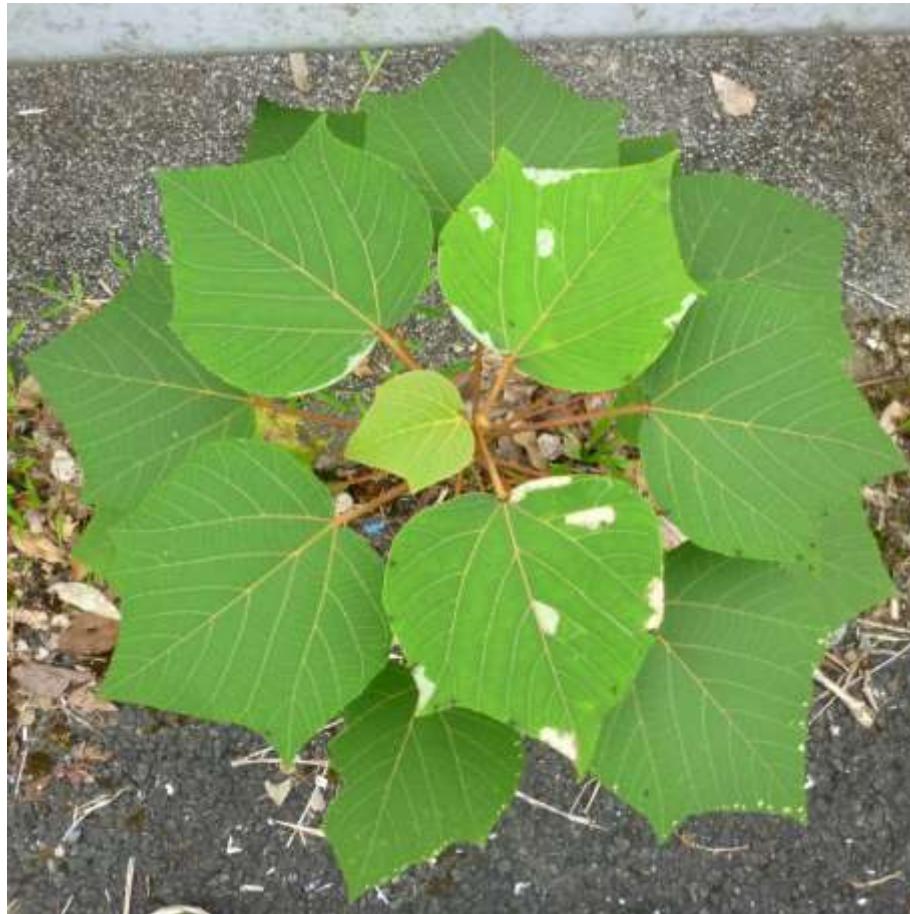
なぜ植物にフィボナッチ数  
が現れるのか

光を効率よく受けるように進化した?

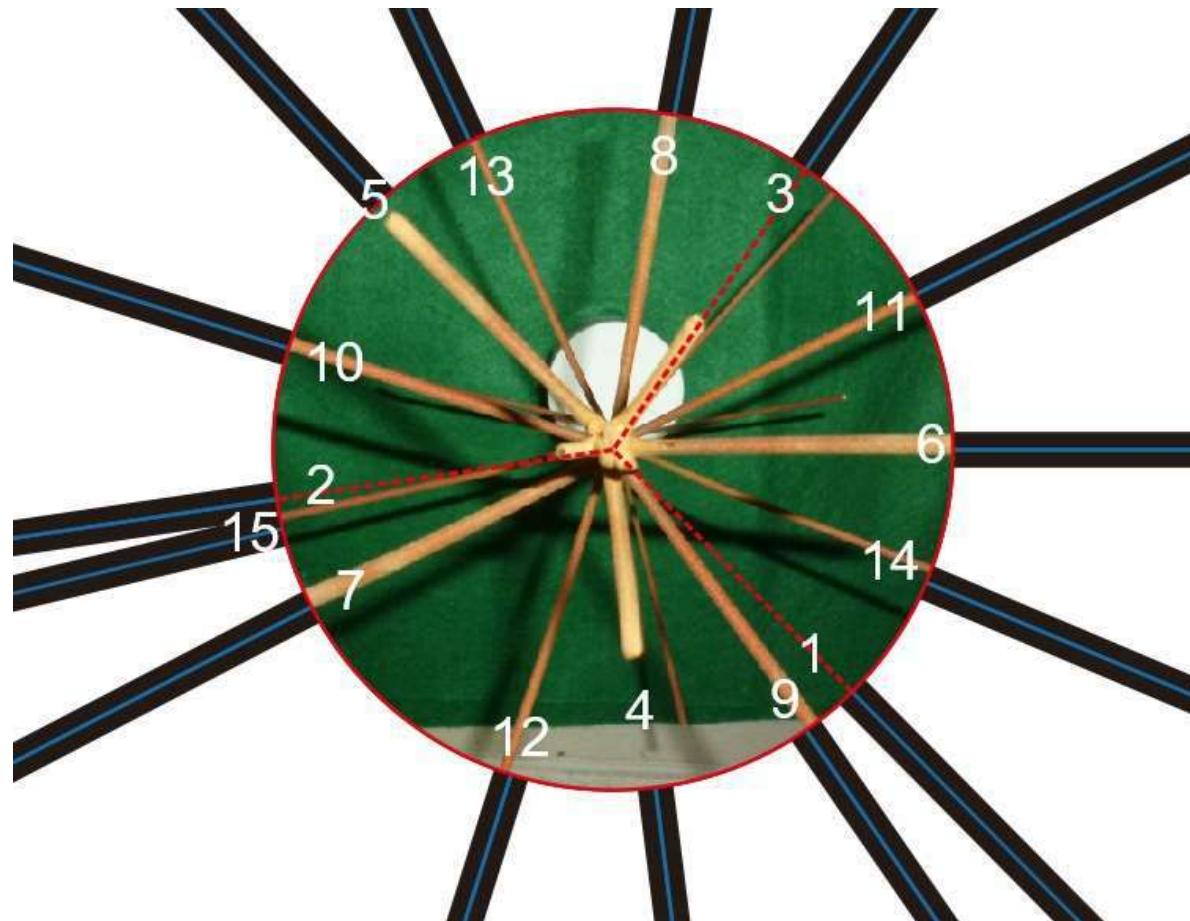
# 葉序 (phyllotaxis)



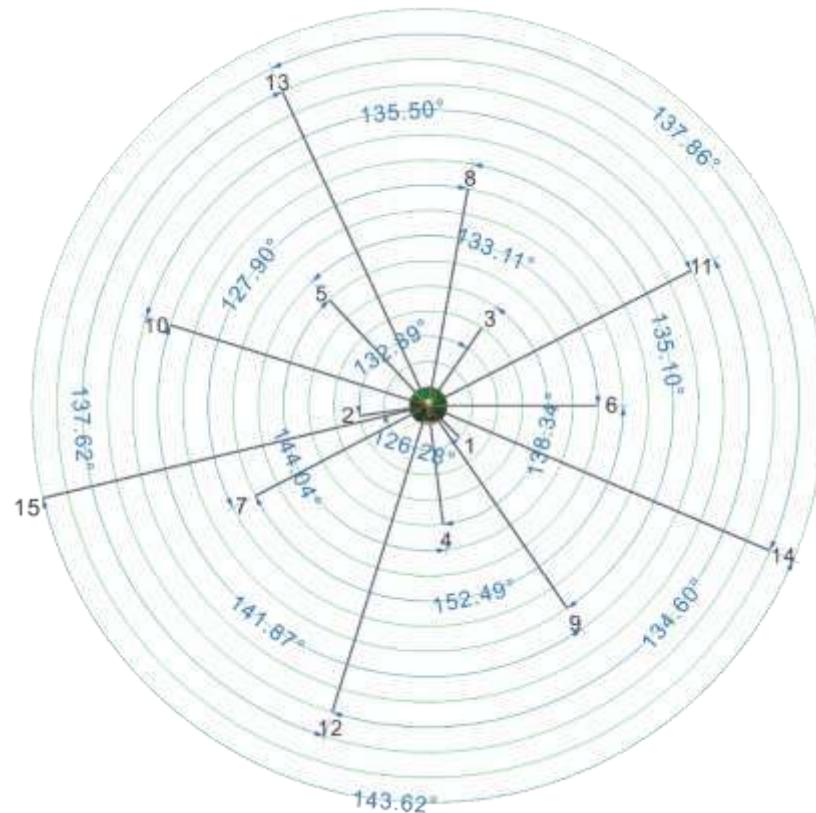
# アカメガシワの幼木



# 葉序(phyllotaxis)の観察

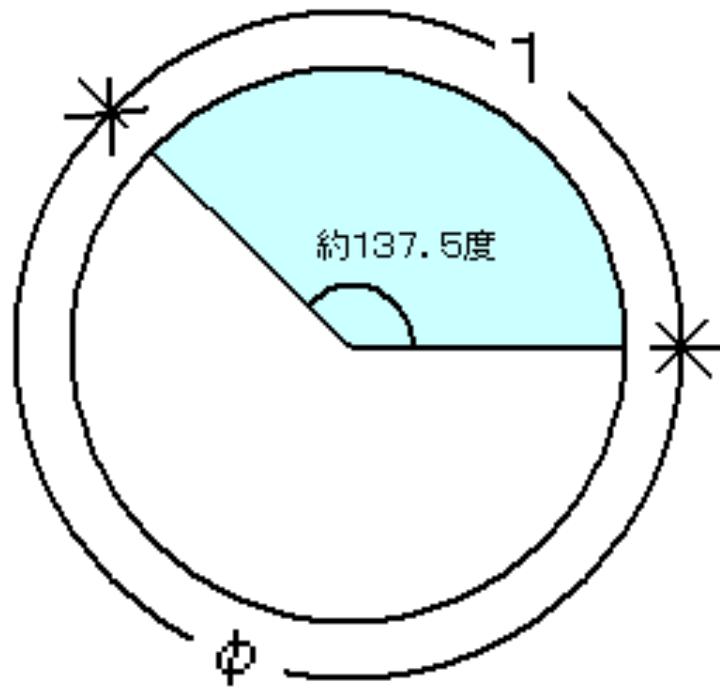


# Schimper-Braunの葉序則



1-2	12628
2-3	13289
3-4	13834
4-5	14404
5-6	13311
6-7	15249
7-8	12790
8-9	13510
9-10	14187
10-11	13550
11-12	13460
12-13	13762
13-14	13786
14-15	14362
平均開度	
13723 度	

# 黄金角 ( $137.5 / 222.5^\circ$ )



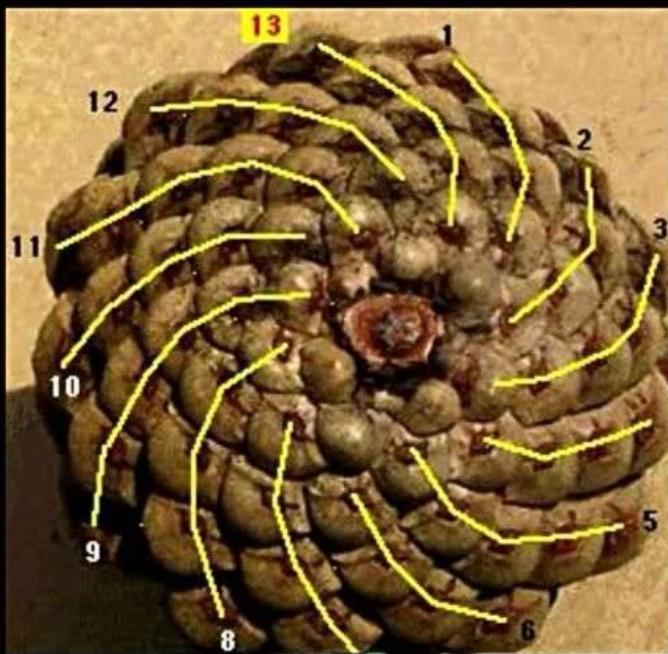
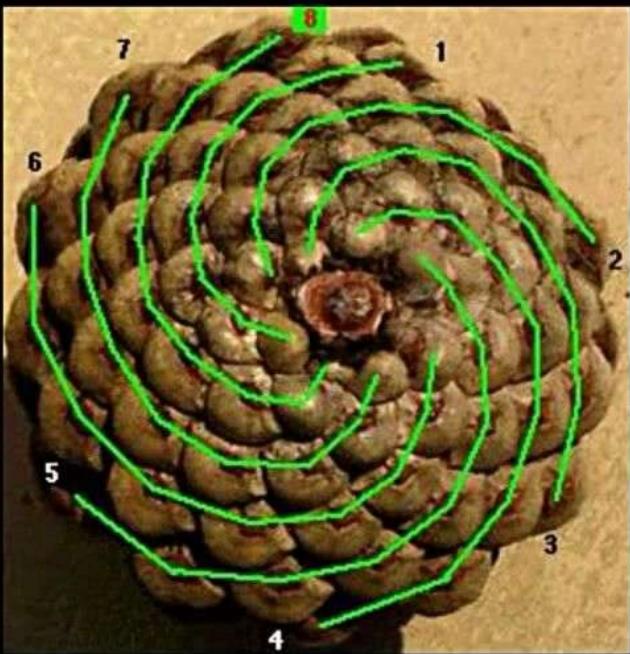
# 黄金角

- 植物では成長するにつれて葉にあたる日光の量が最大になるように、葉を茎にうまく配置する必要がある。上下の葉がぴったり重なつていたら、日光がまったくあたらなくなってしまうからである。
- 最善の配置をもたらす角度は $360 \times 1/(1+\varphi)=137.5^\circ$  とされる。この角度は $360^\circ$  を $1:\varphi$ に黄金分割したものになっていて「黄金角」と呼ばれる。

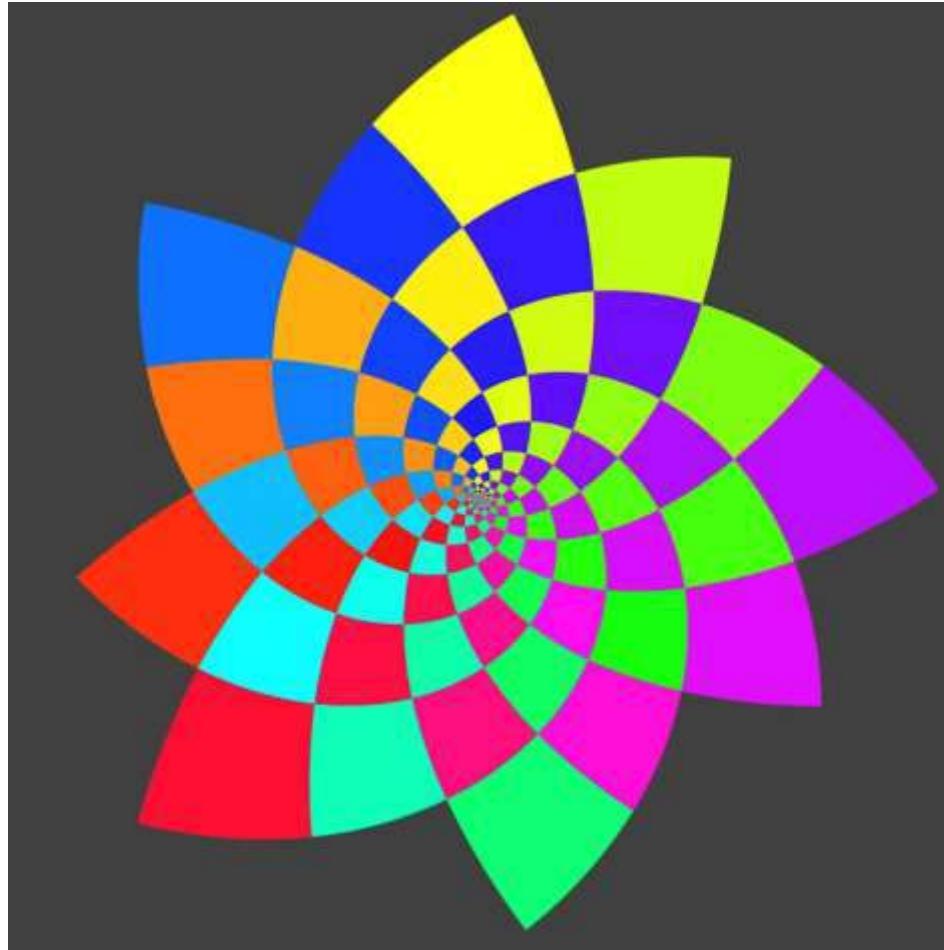
# 黄金角

- 植物では成長するにつれて葉にあたる日光の量が最大になるように、葉を茎にうまく配置する必要がある。上下の葉が重なったり重なついたら、光がたくさんあたらなくなってしまうからである。
- 最善の配列をもたらす角度は $360 \times 1/(1 + \sqrt{5}) = 137.5^\circ$ となる。この角度は $360^\circ$ を $1:\varphi$ に分ける黄金比の角になっていて「黄金角」と呼ばれる。

# 松毬のparastichy (8/13)



# Fibonacci tiling (8/13)



# ヒマラヤ杉



# ヒマラヤ杉



# ヒマラヤ杉



# Fibonacci Tree, collaboration with Image Mission Inc.

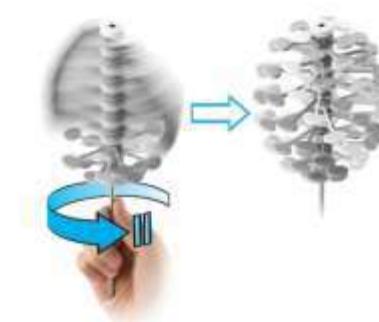
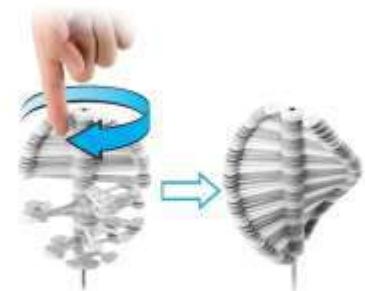


The creator of  
Fibonacci Tree:

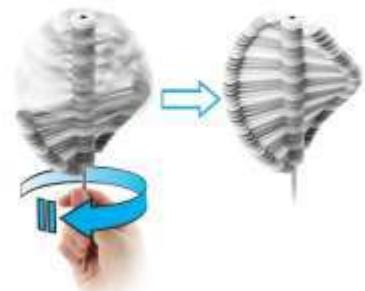
John Edmark

## How to Play

- 1 Manually rotate all petals clockwise to form a helix shape.



- 2 Quickly twist the handle in counter-clockwise direction, and hold until Lollipopter transforms into a pinecone shape.



- 3 Twist in reverse direction to transform back to a helix shape.

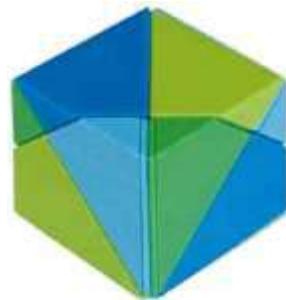
# Image Mission Inc.

(株式会社イメージミッション木鏡社)

- What does Image Mission Inc. do?
  - A Japanese company based in Shizuoka.
  - Make, Import & Sell scientific and educational toys.
- Other products of Image Mission Inc.



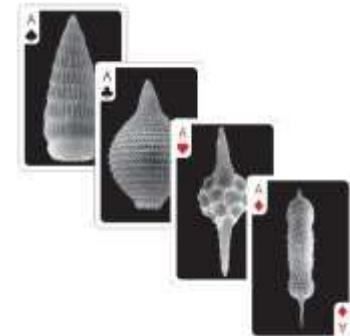
Imaginary Cube Puzzle



Pentadron



GeoBalls



Radiolaria Cards

Please look at web page for more information of  
Image Mission Inc. Thank you!



[www.imagemission.com](http://www.imagemission.com)

# Fibonacci spiral (21/34)



# Fibonacci spiral (34/55)



らせんの形状を変化させて  
原点付近での振る舞いを調べる

ヒマワリのらせんを数学的に再現

# 代表的な平面らせん

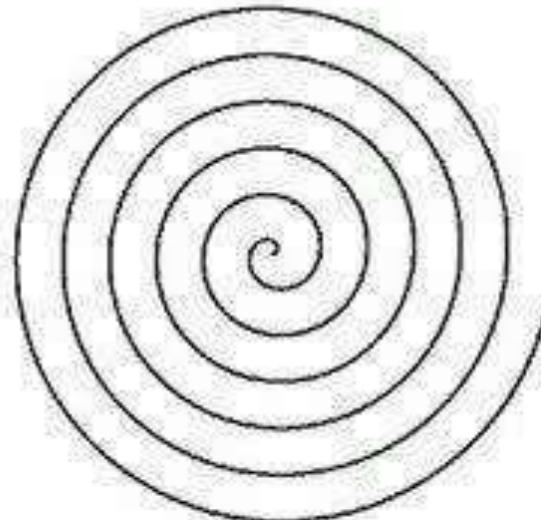
- 代表的な平面らせん

[a] Bernoulliらせん :  $r=a^\theta$

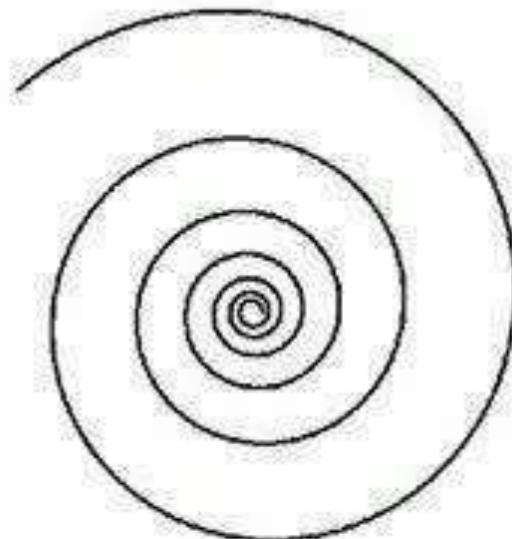
[b] Archimedesらせん :  $r=a\theta$

[c] Fermatらせん :  $r^2=a\theta$

では、隣接する渦巻きの間隔は原点から遠いほどそれぞれ広くなる・等間隔・狭くなる

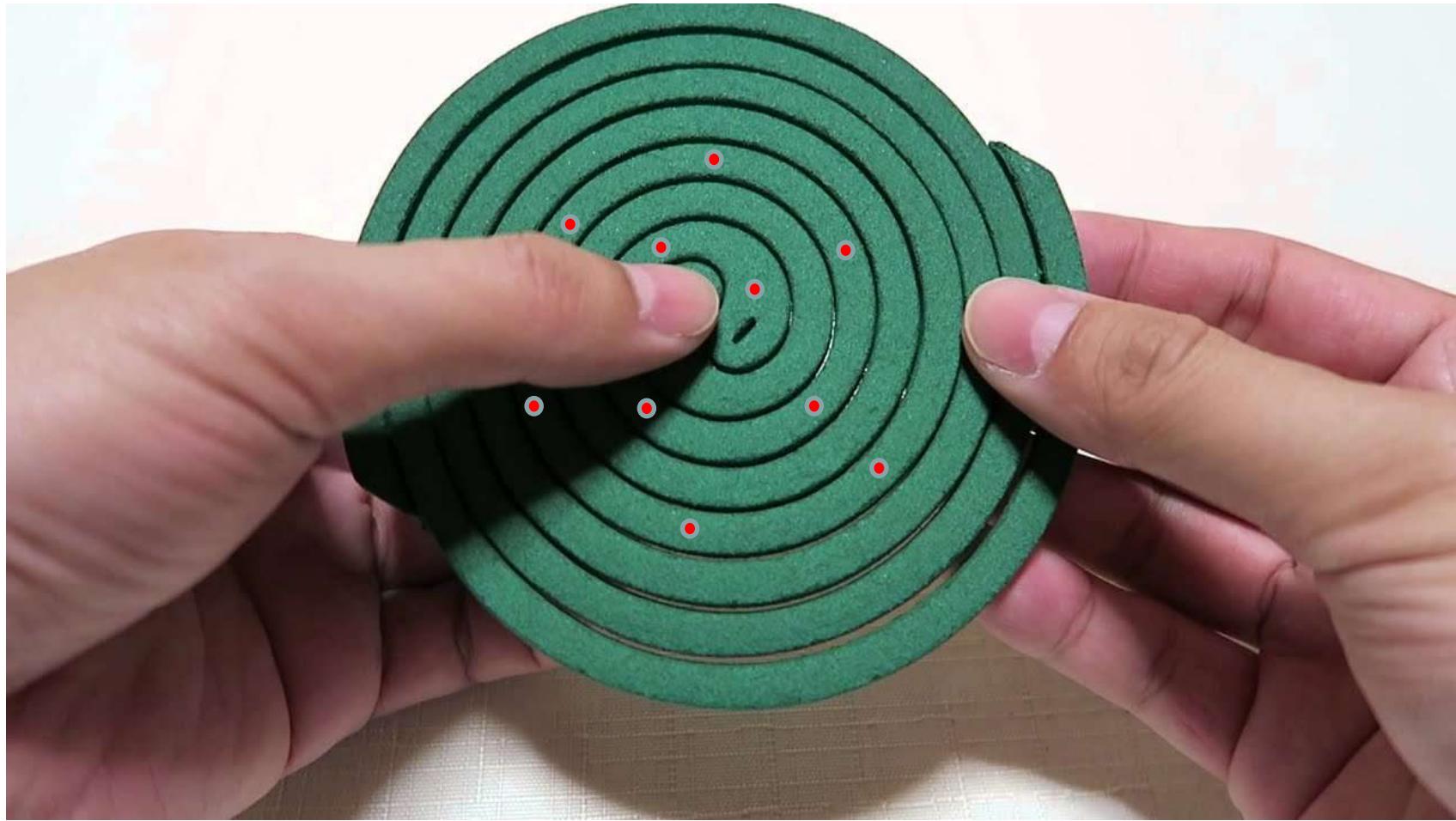


アルキメデス螺旋



対数螺旋

$$(r_n, \theta_n) = (n, 2\pi n\alpha), \alpha(0, 1)$$



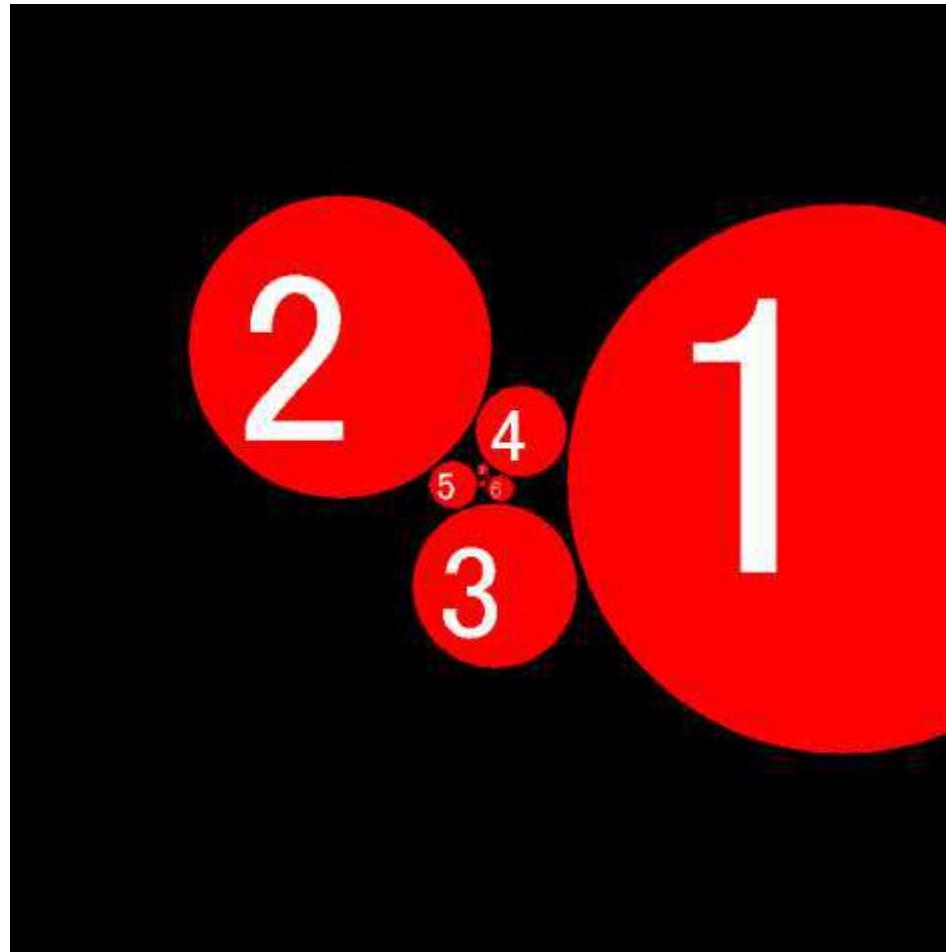
## Question asked by a high school student

- ヒマワリの花は外側に向かって構成されるのか、内側から順次構成されるのか？

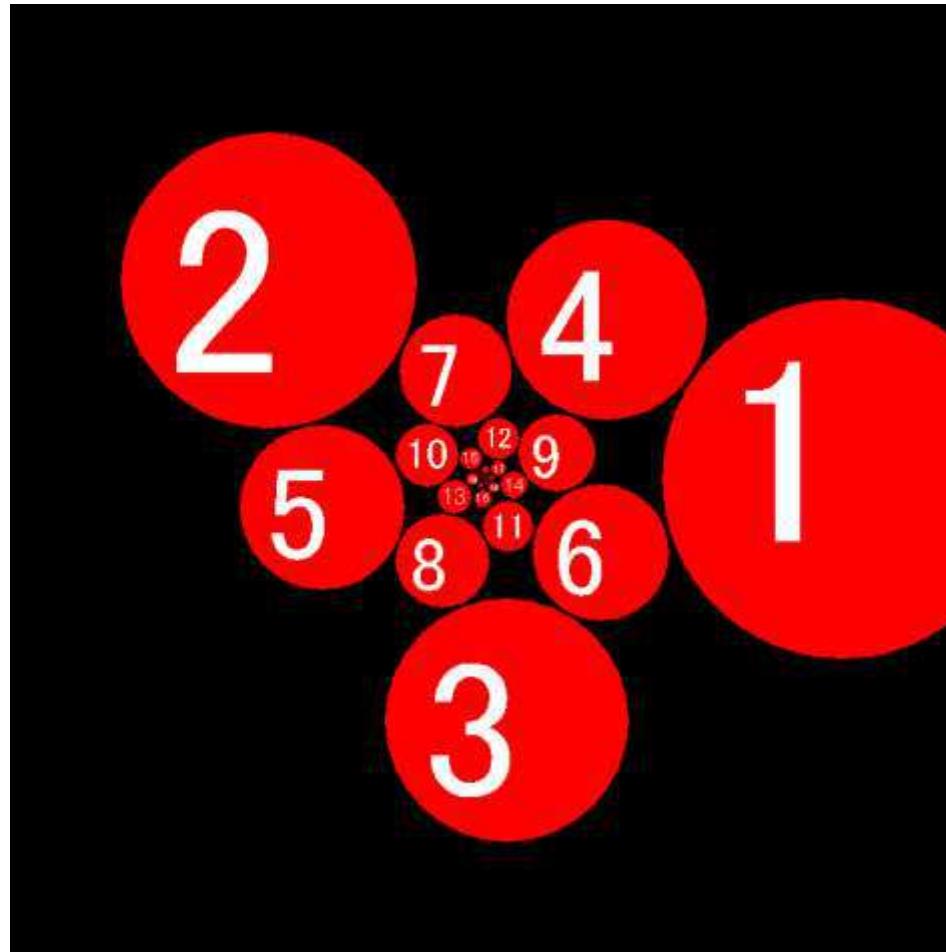
# A. 原基は渦の中心にあり



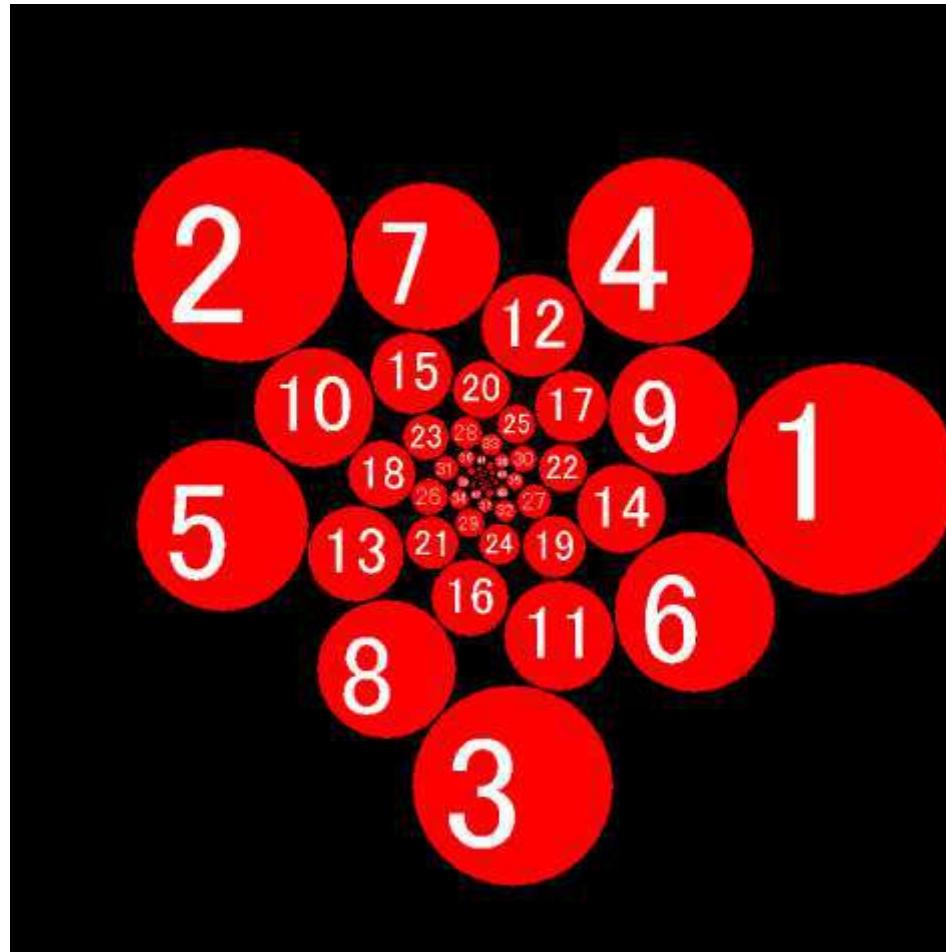
# Fibonacci numbers (2/3)



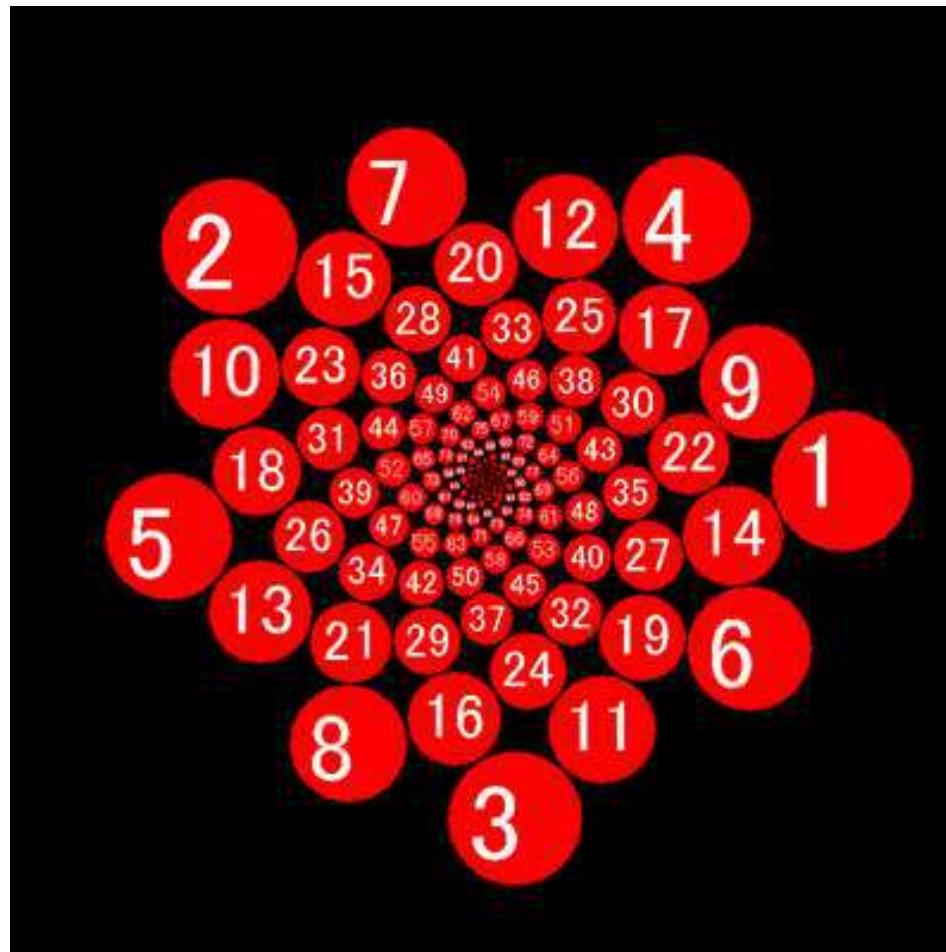
# Fibonacci numbers (3/5)



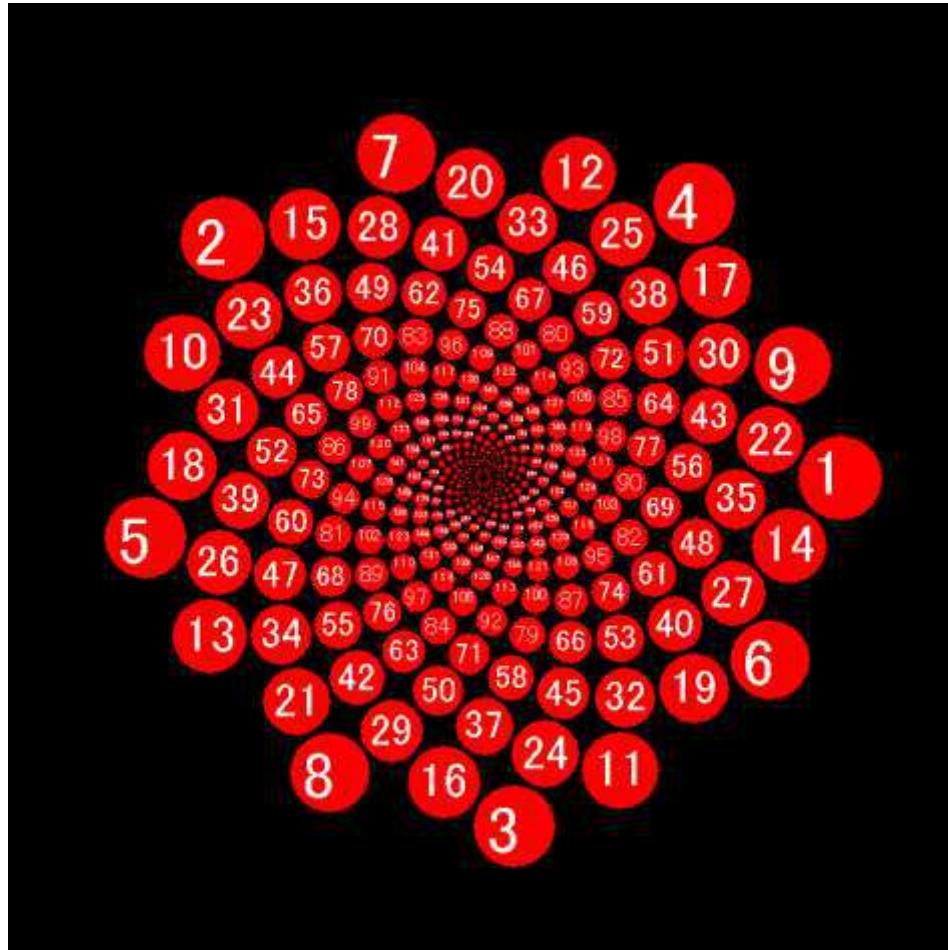
# Fibonacci numbers (5/8)



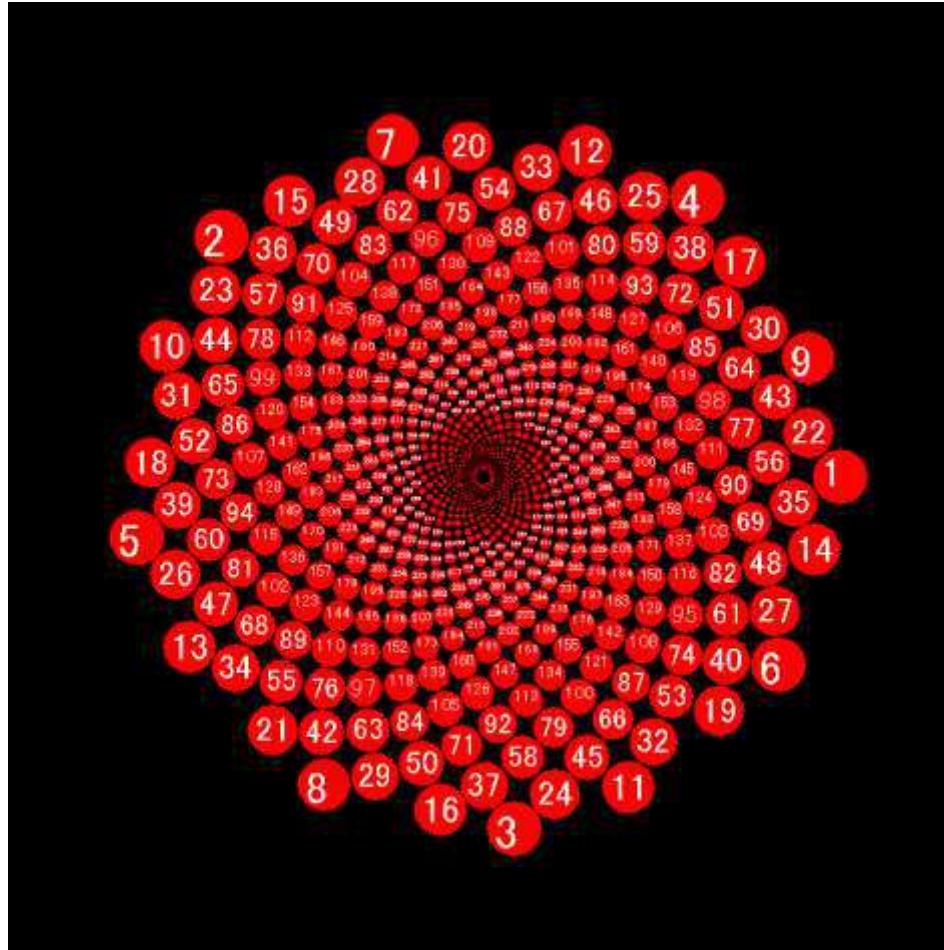
# Fibonacci numbers (8/13)



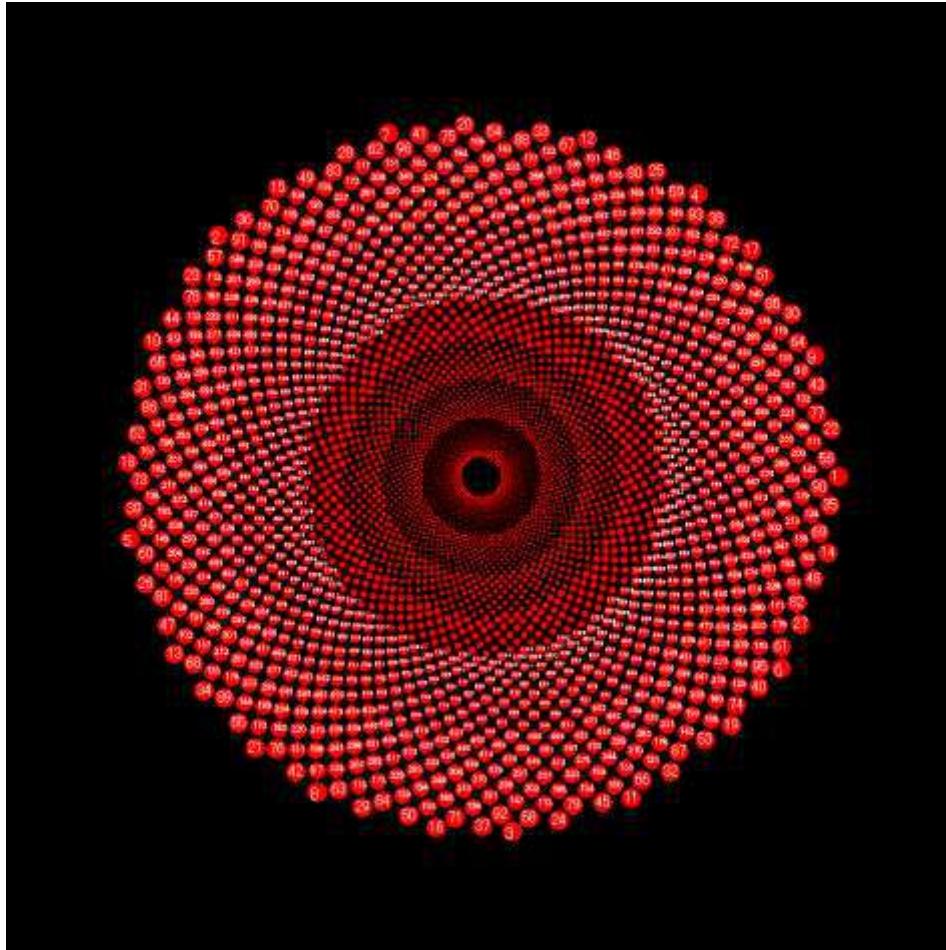
# Fibonacci numbers (13/21)



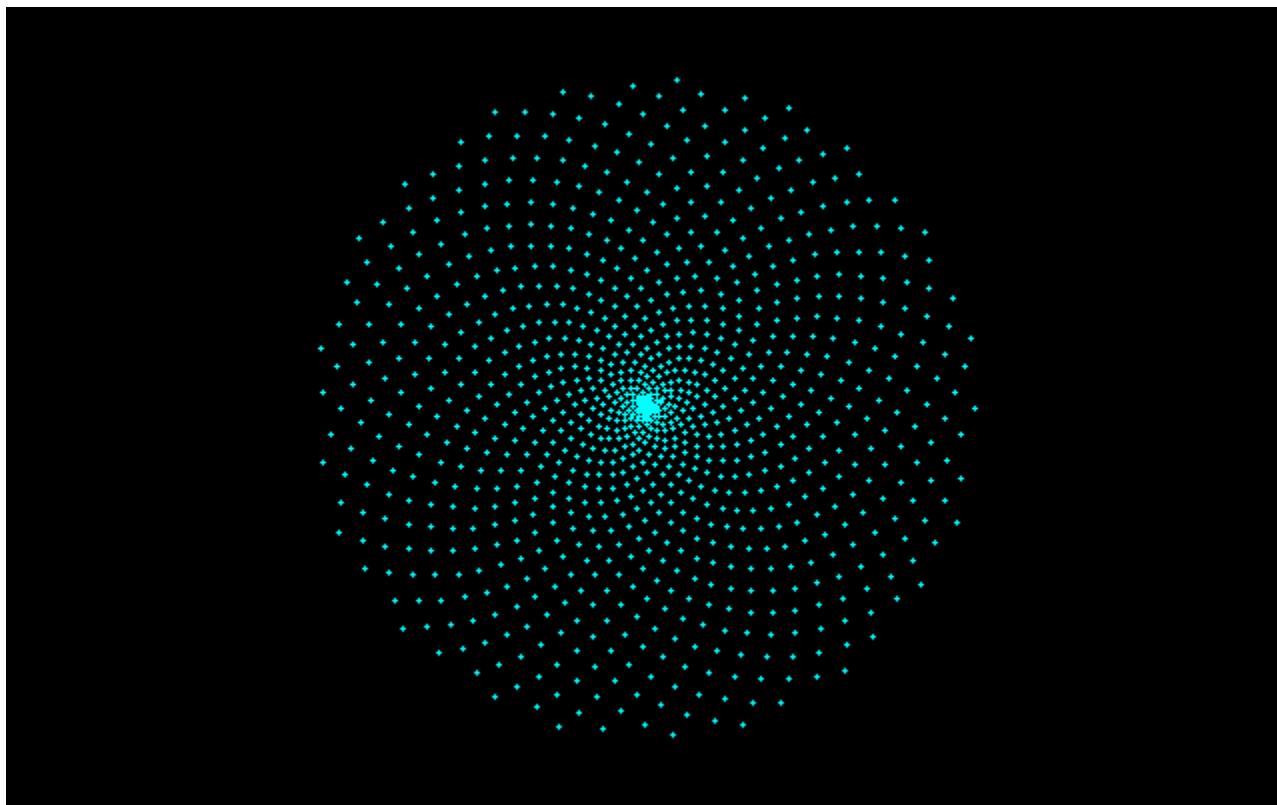
# Fibonacci numbers (21/34)



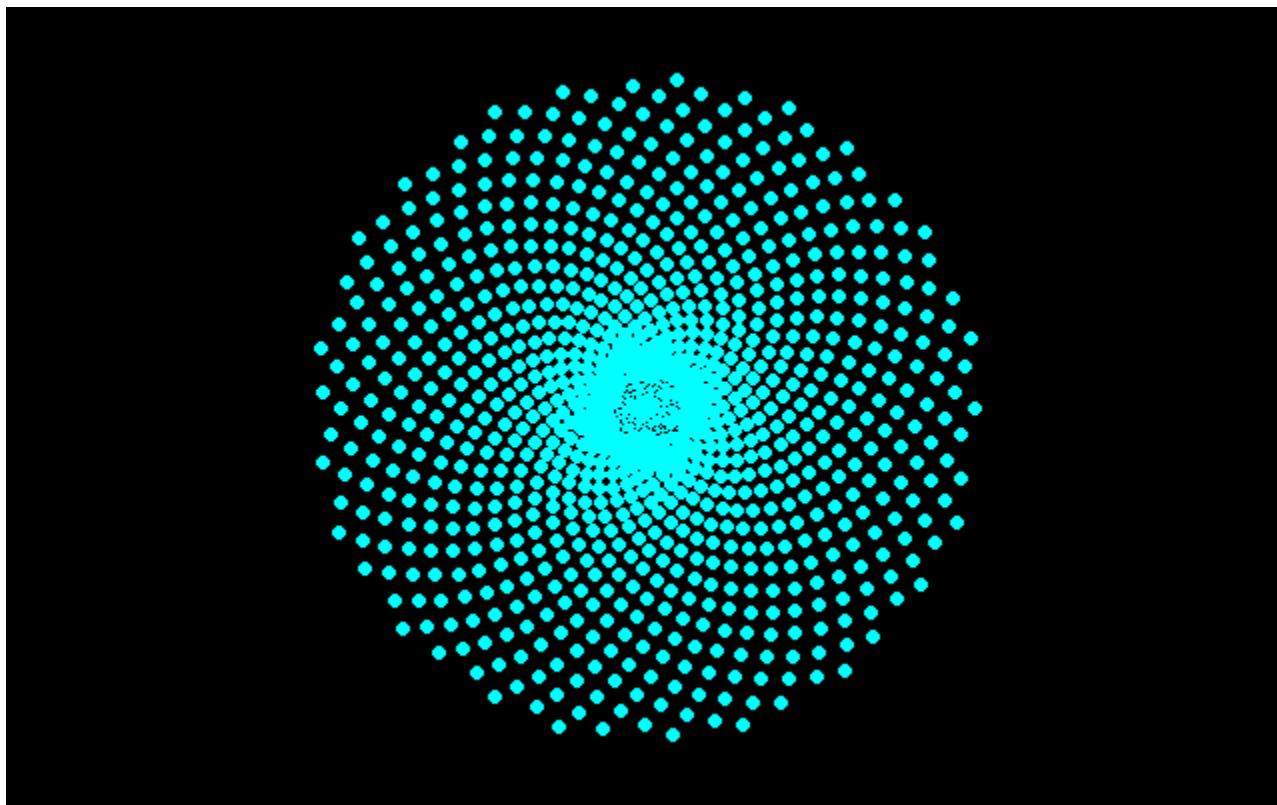
# Fibonacci numbers (34/55)



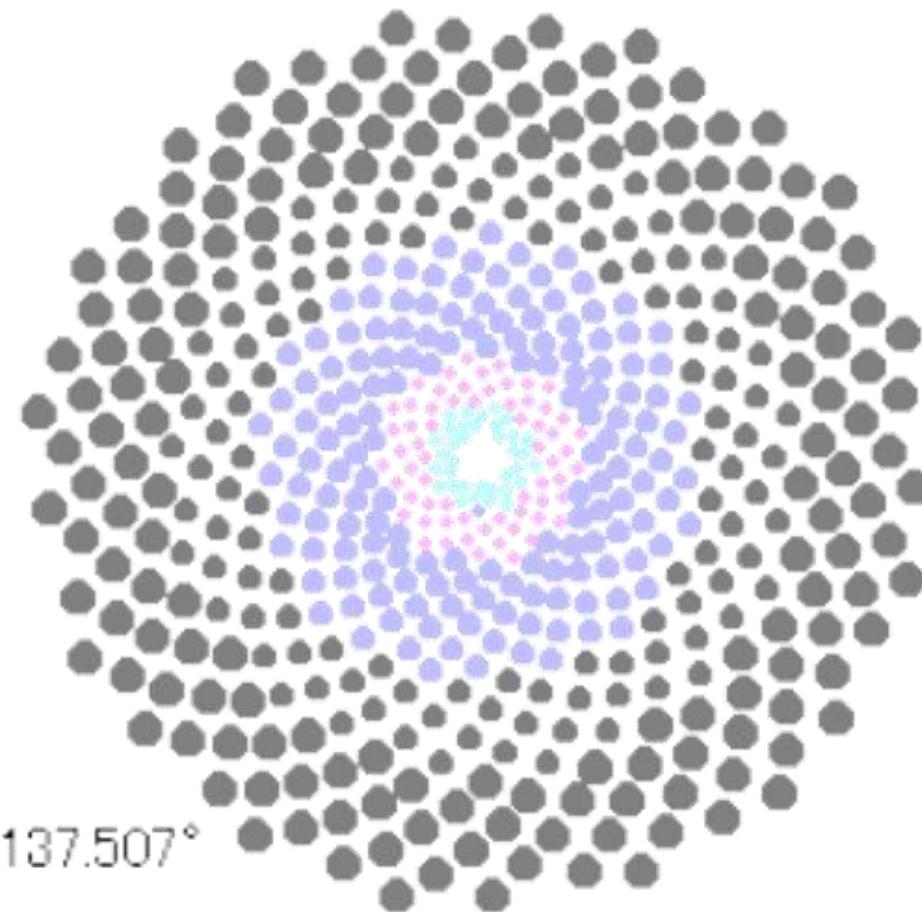
$\alpha=137.5^\circ / 222.5^\circ$



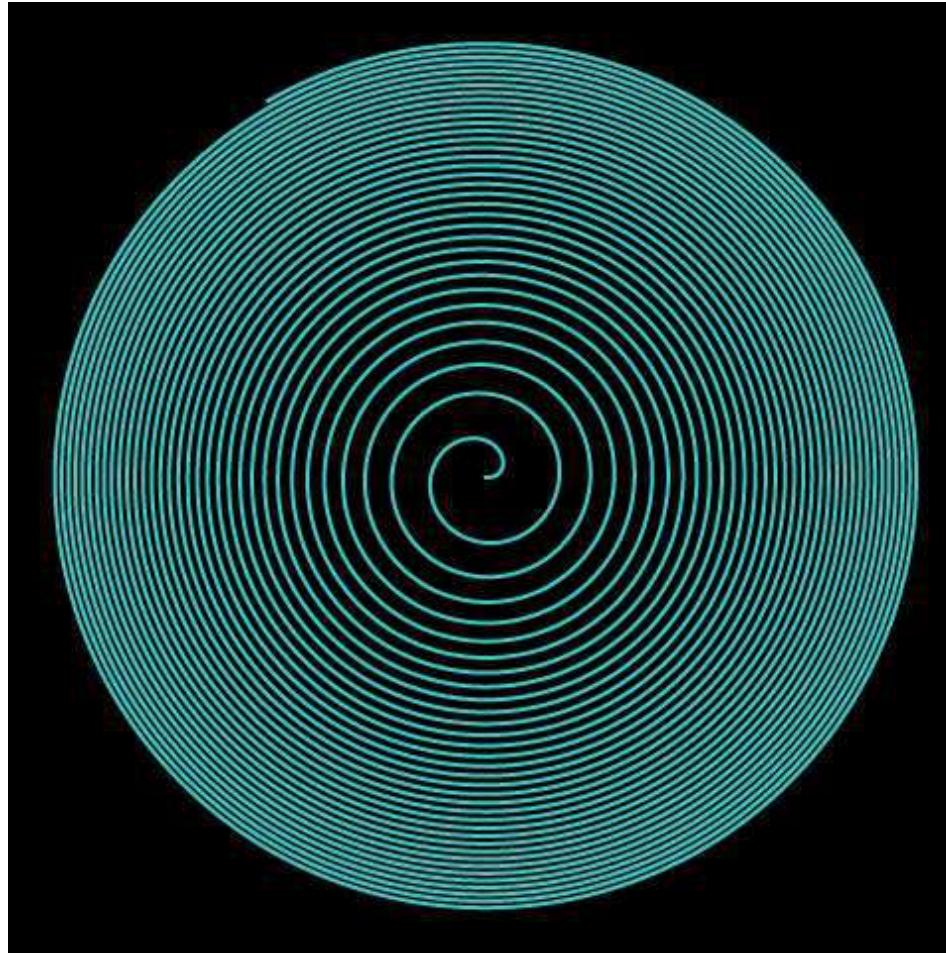
$\alpha=137.5^\circ / 222.5^\circ$



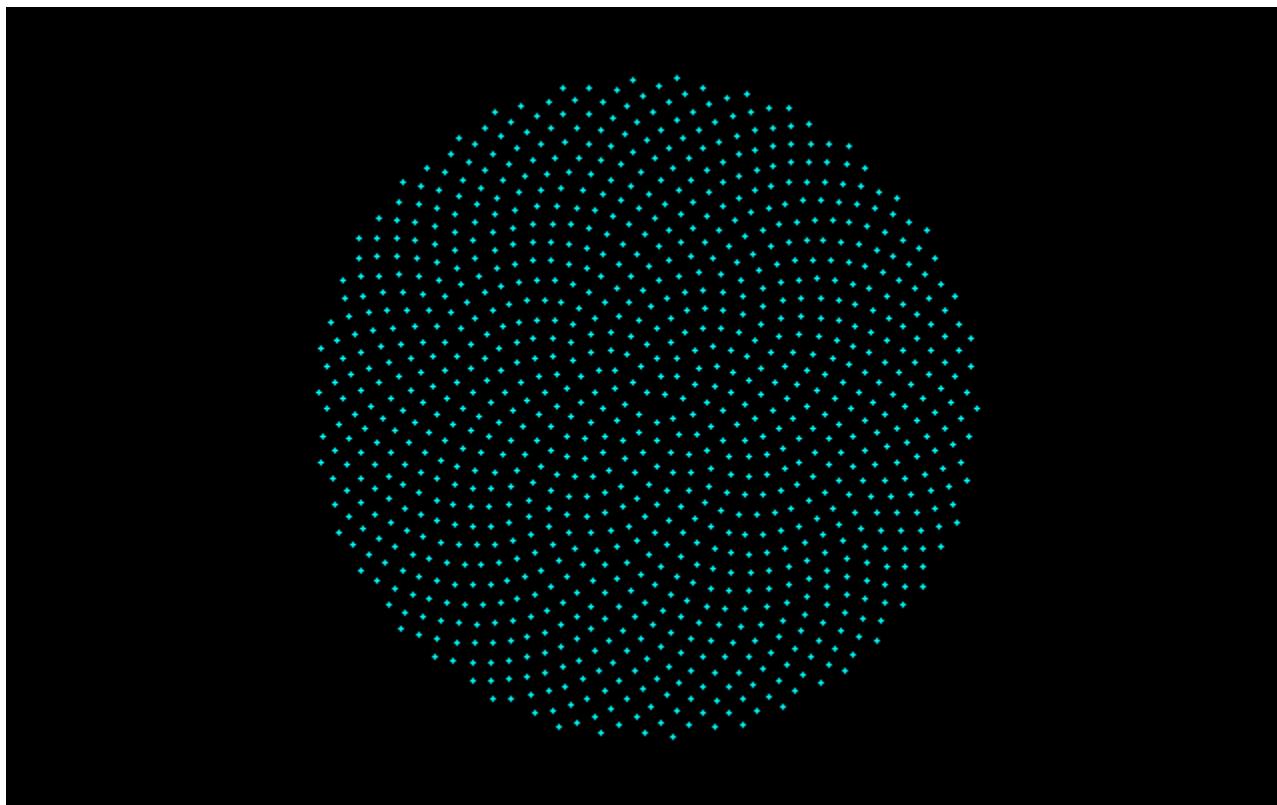
# アルキメデスらせん上の点配置



$$(r_n, \theta_n) = (\sqrt{n}, 2\pi n\alpha), \alpha(0, 1)$$

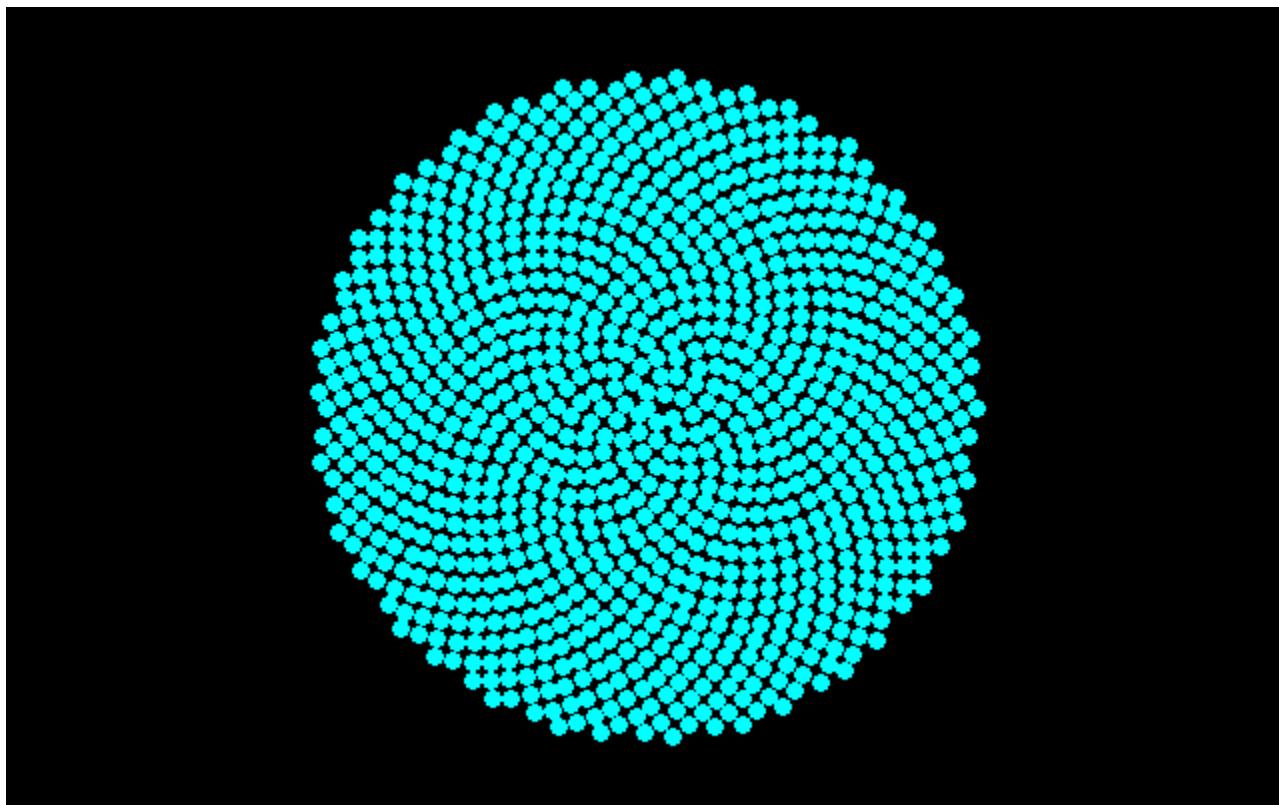


$$\alpha = 1/\varphi = (\sqrt{5}-1)/2$$

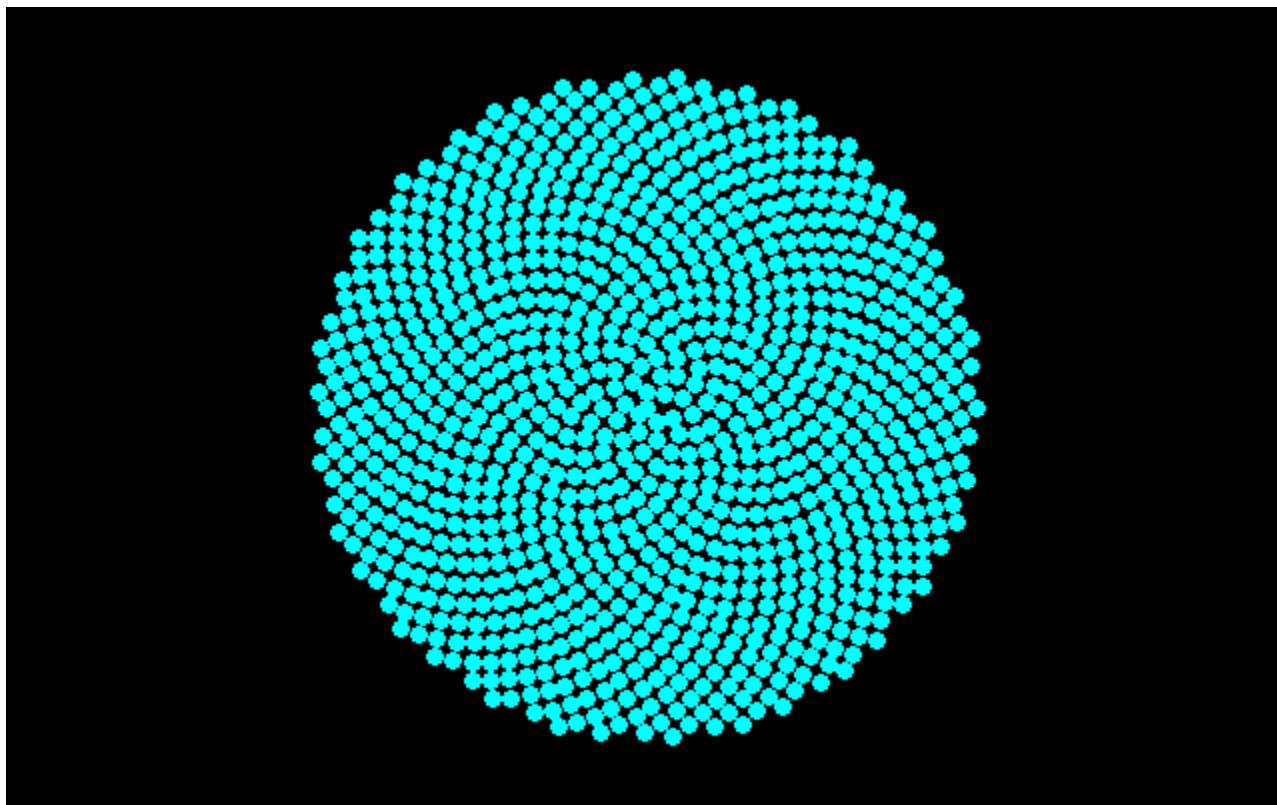


222.5

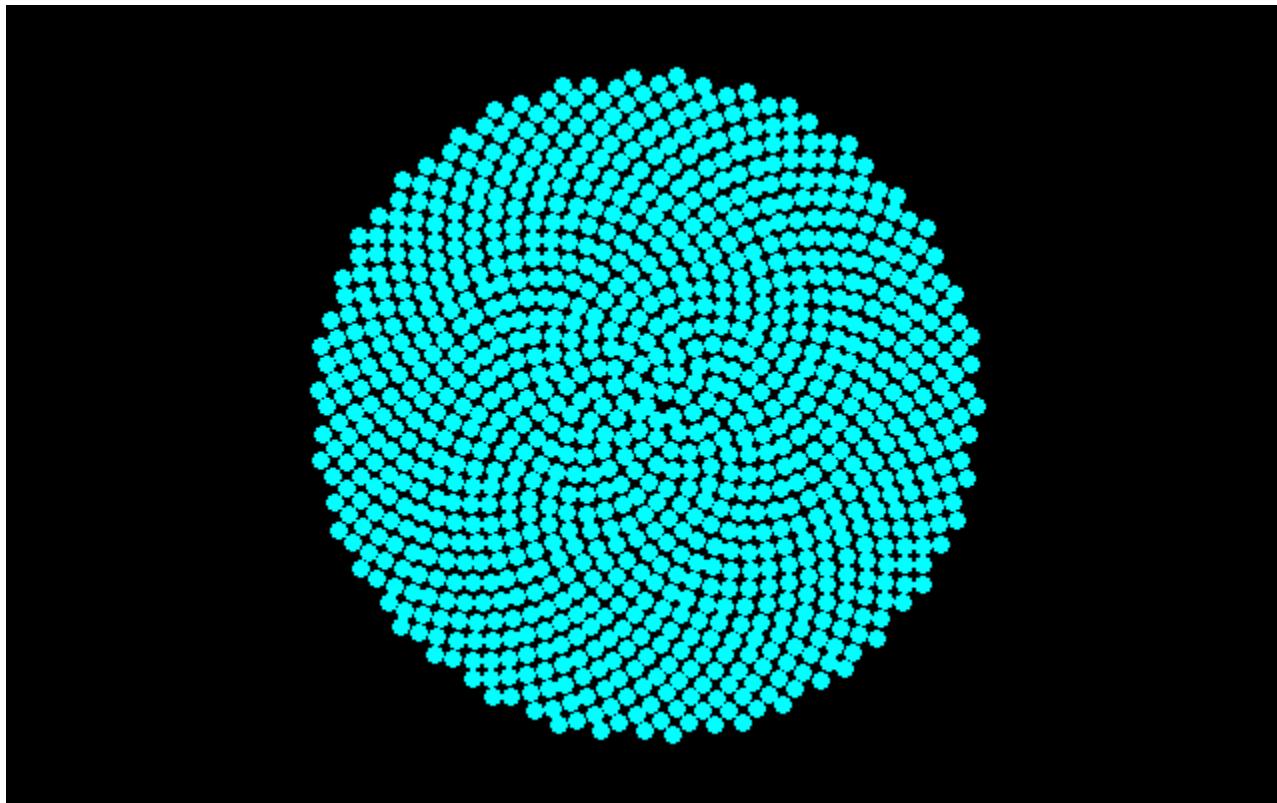
# Voronoi領域の面積がほぼ一定



# 最も効率の良い配置



# 準結晶 (Delone set)



# らせん上の点配置

- 黄金角とは $360^\circ$ を $1:\varphi$ に内分した角である  
( $137.5 / 222.5^\circ$ )
- 成長期はアルキメデスらせん( $r=a\theta$ )
- 成熟期はフェルマーらせん( $r^2=a\theta$ )
- 実際はアルキメデスらせんとフェルマーらせんの中間的ならせん上の点配置 ( $r^{1.5}=a\theta$ )?
- 成長につれてらせんが変化する?

# 黄金角の仲間たち

- 黄金角:  $360 \times 1/(1+\varphi) = 137.5^\circ$
- 黄金角:  $360 \times \varphi/(1+\varphi) = 222.5^\circ$
- Lucas角:  $360 \times 1/(2+\varphi) = 99.5^\circ$
- ?????角:  $360 \times 1/(4-\varphi) = 151.1^\circ$
- ?????角:  $360 \times 1/(5-\varphi) = 106.4^\circ$

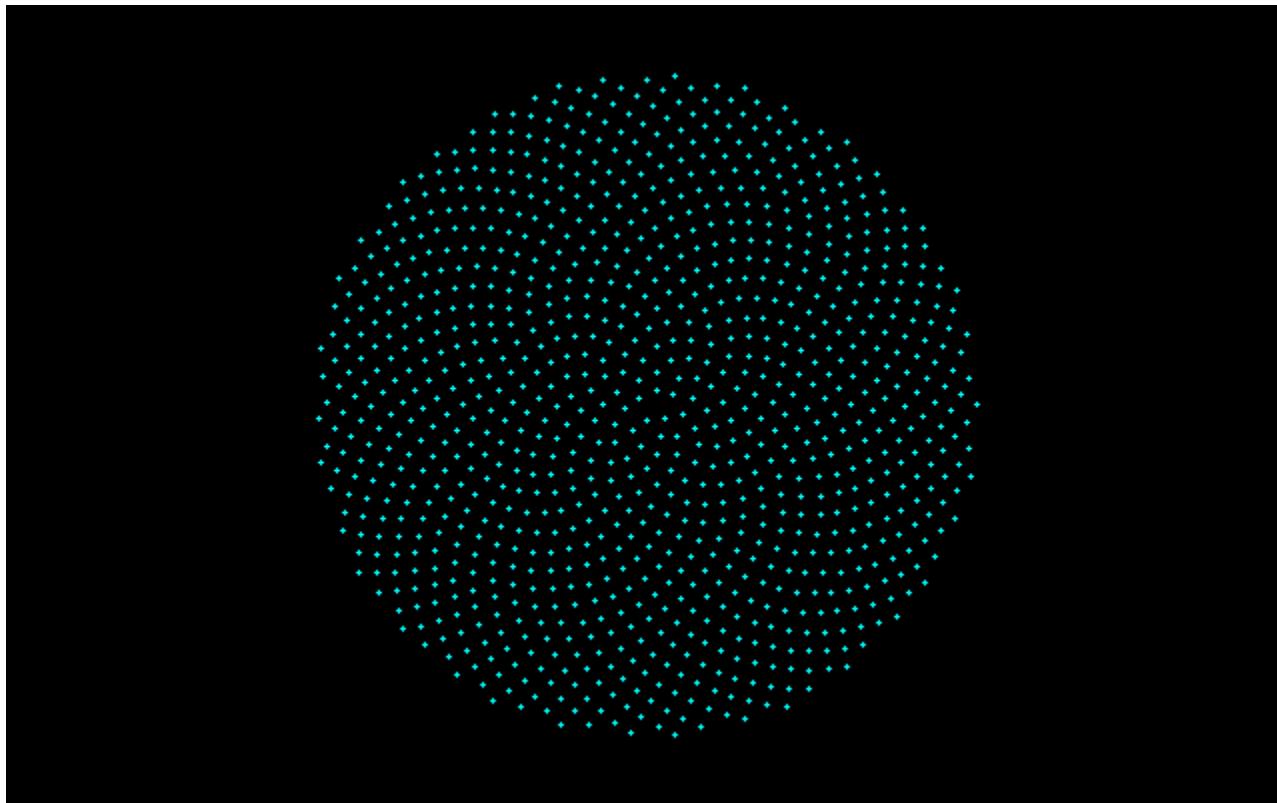
The Lucas divergence angle =  $99.5^\circ$ . This is  $360/(1 + \Phi^2)$ .

This is found in 1.5% of observed phyllotaxis plant patterns including:

some sunflowers, certain cedars, sequoias, balsam trees and other species.

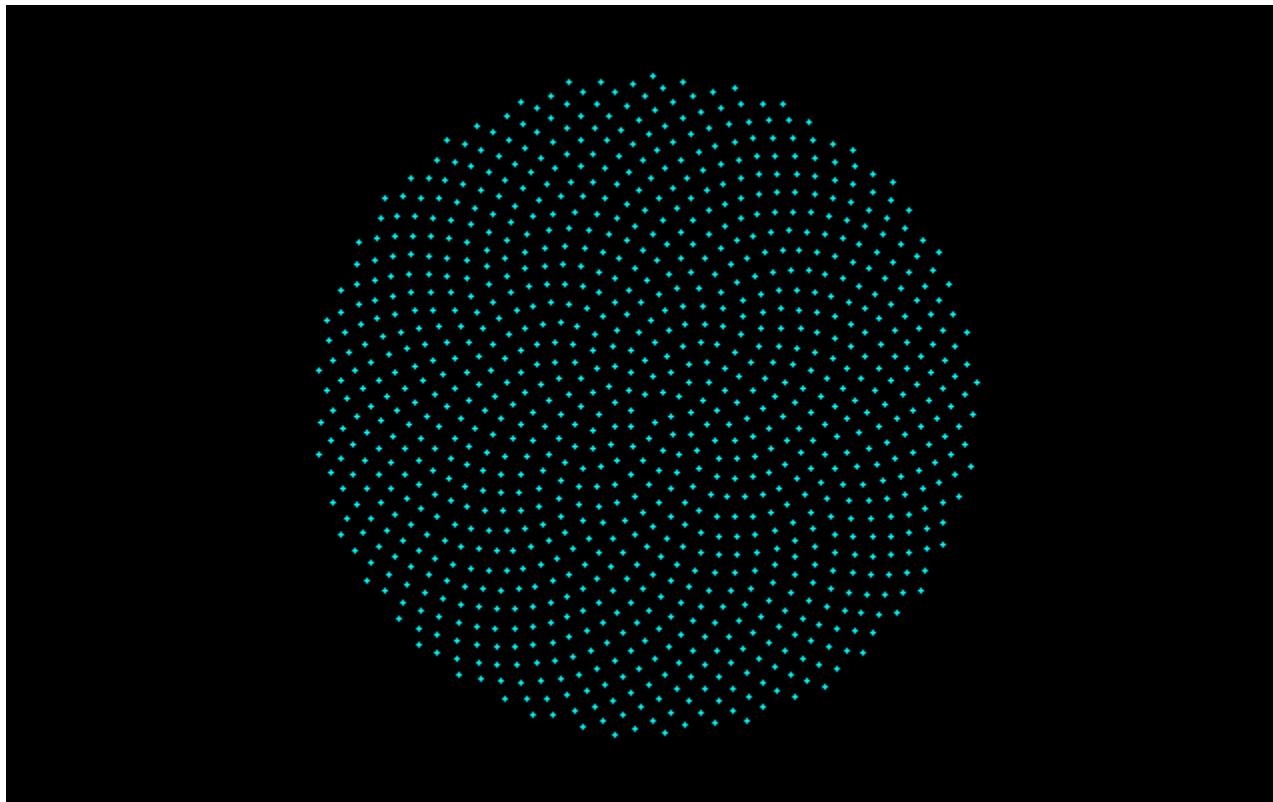


黃金角,  $\alpha=1/(\varphi+1)$   
=[0;2,1,1,1,1,1,1,1,1,· · ·]



137.5

Lucas角,  $\alpha=1/(\varphi+2)$   
=[0;3,1,1,1,1,1,1,1,1, $\cdots$ ]



99.5

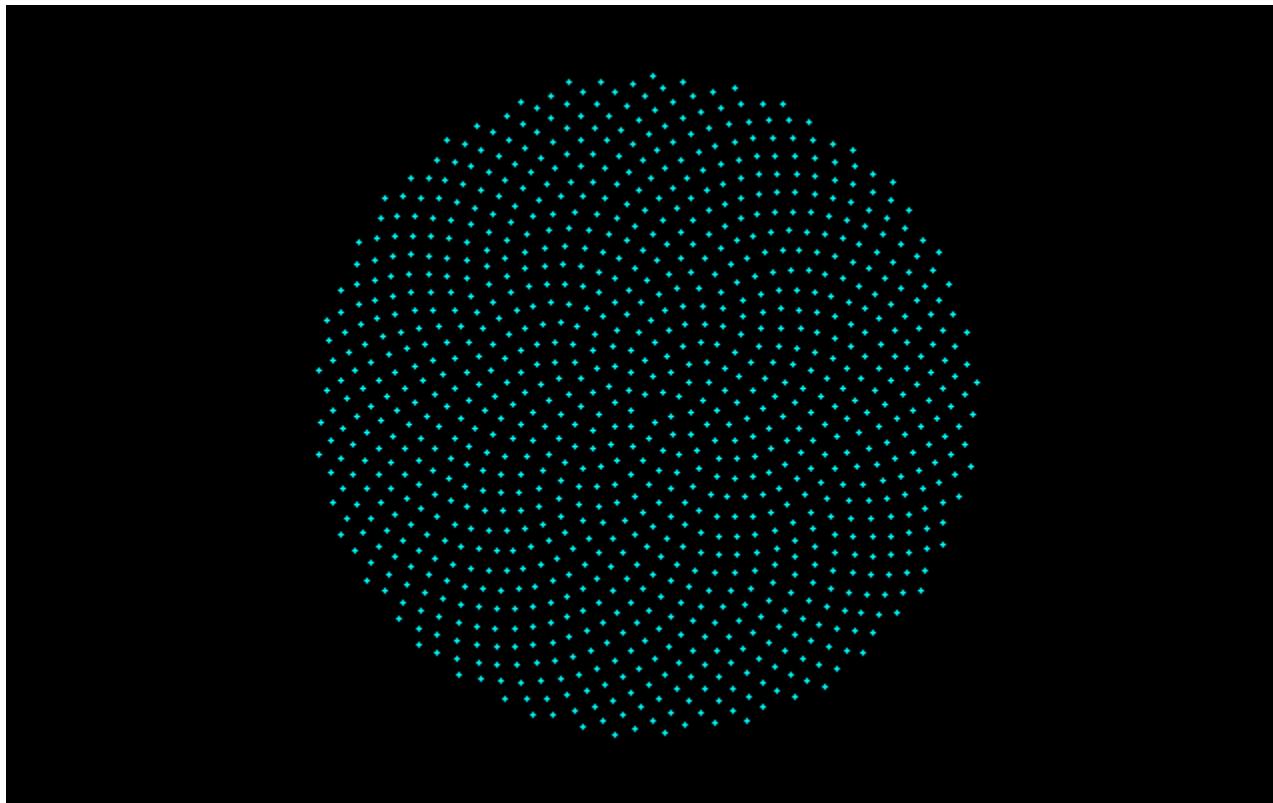
# Question asked by a high school student

- 黄金角とリュカ角の密度はほとんど変わらないので、ぱッと見分けるのは難しい。
- それでは何をもって、ヒマワリの形の特徴とするのか？

# A. ヒマワリの形の特徴

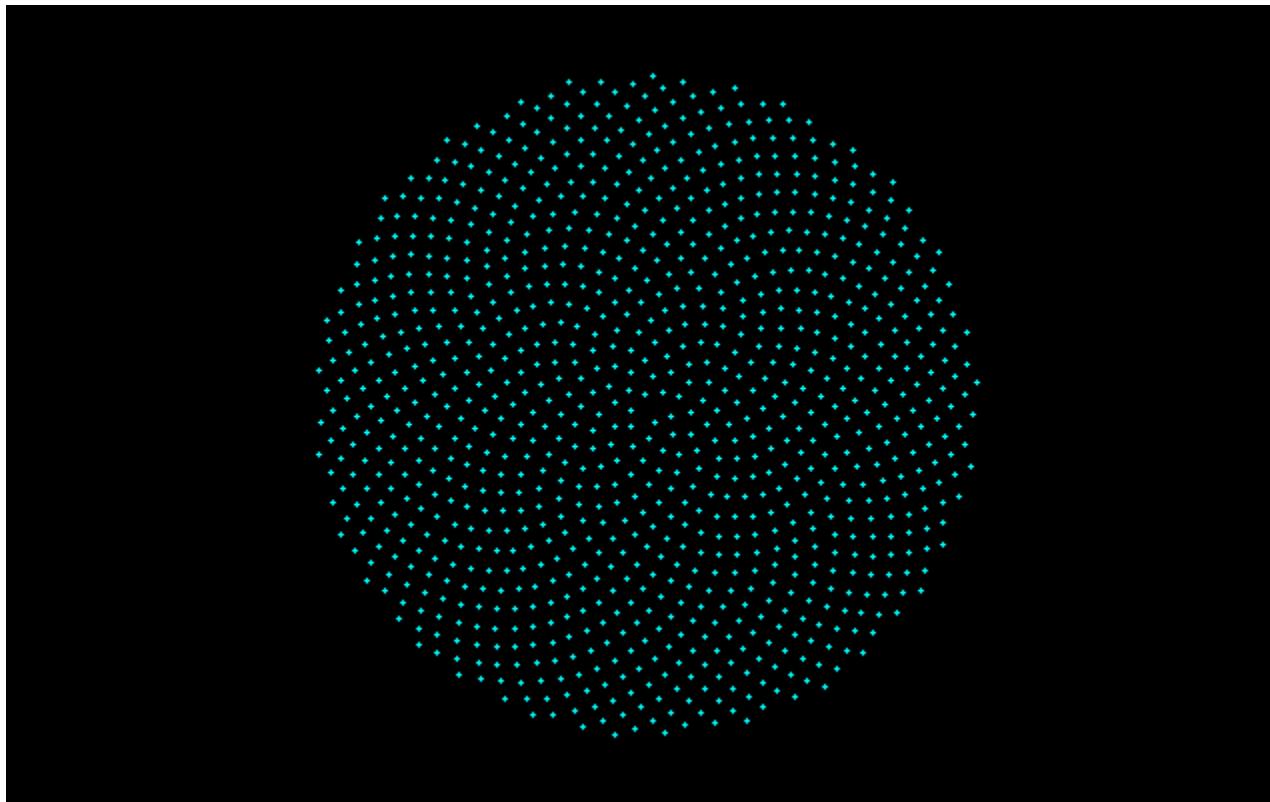
- ・ヒマワリの形の特徴は、時計回りの渦巻きと反時計回りの渦巻きが**錯綜配列・交錯**することである
- ・開度 $\alpha$ を変化させて、ヒマワリパターンが現れる開度 $\alpha$ がどのように分布するかを調べてみよう。その際、開度 $\alpha$ の**連分数表現**を併記する

Lucas角,  $\alpha=1/(\varphi+2)$   
=[0;3,1,1,1,1,1,1,1,1, $\cdots$ ]



99.5

Lucas角,  $\alpha=1/(\varphi+2)$   
=[整数部分;部分商]



開度

連分数展開,  $\varphi = [1; 1, 1, 1, 1, 1, \dots]$

$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}$$

連分数展開,  $\varphi = [\text{整数}; \text{部分商}]$

$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}$$

$$\varphi = [1; 1, 1, 1, 1, 1, \dots] = 1.618$$

$$\varphi - 1 = [0; 1, 1, 1, 1, 1, 1, \dots] = 0.618$$

$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}$$

# $\alpha$ の連分数展開

$$\sqrt{2} = [1; 2, 2, 2, 2, 2, 2, \dots]$$

$$\sqrt{3} = [1; 1, 2, 1, 2, 1, 2, \dots]$$

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\dots}}}}}}$$

$$\sqrt{3} = 1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{\dots}}}}}}$$

# Expansion into continued fraction

$$\alpha = [a_0; a_1, a_2, a_3, \dots]$$

$$\pi = [3; 7, 15, 1, 292, 1, 1, \dots]$$

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{\dots}}}} = [a_0; a_1, a_2, a_3, \dots]$$

$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}$$

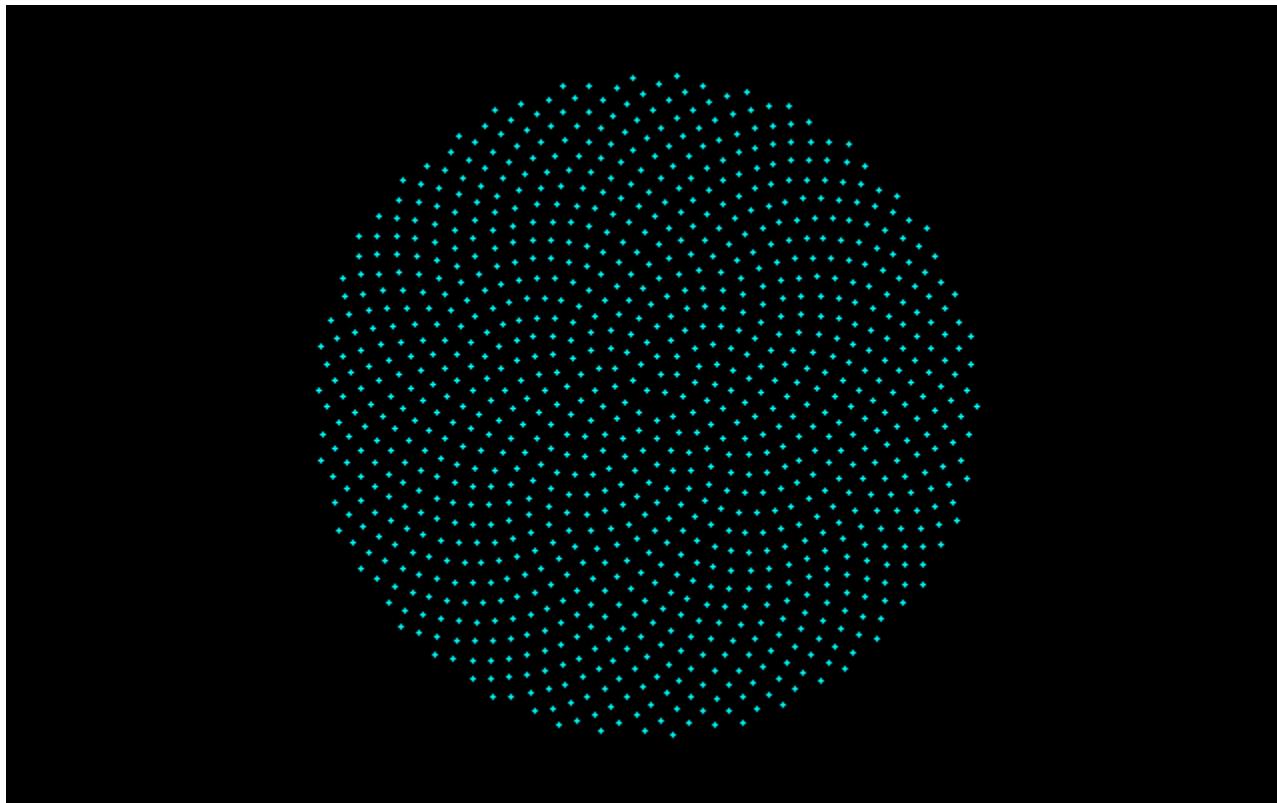
開度 $\alpha$ をダイナミックに変化させて  
原点付近での振る舞いを調べる

ヒマワリのらせんを数学的に再現

# ヒマワリのらせんを数学的に再現

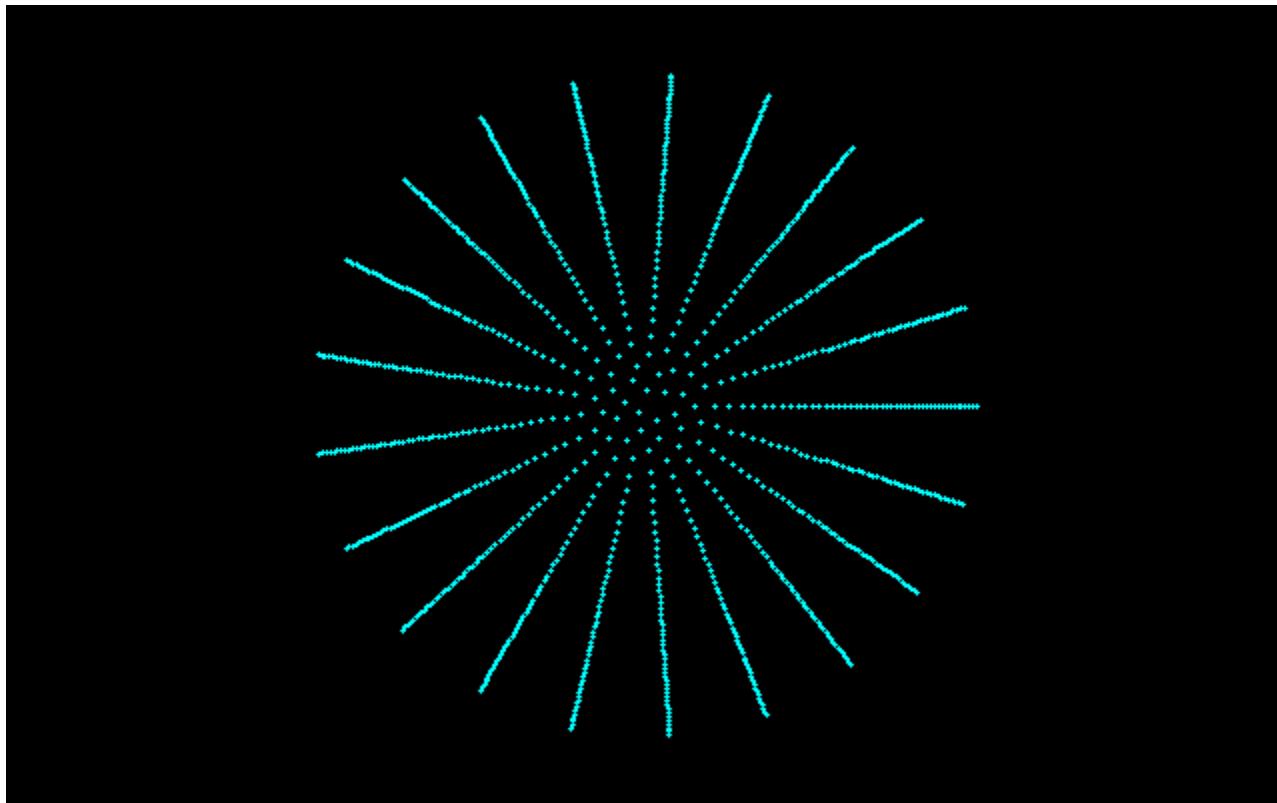
- フェルマーらせん
- 開度 $\alpha$ は一定とする
- フィボナッチパターンが現れるのは黄金角のときに限るとされている。成長につれて開度 $\alpha$ が変化する可能性もあるが…

$$1/\varphi = (\sqrt{5}-1)/2 = [0; 1, 1, 1, 1, 1, 1, \dots]$$



222.5

$$\alpha = 13/21 = [0; 1, 1, 1, 1, 1, 2]$$

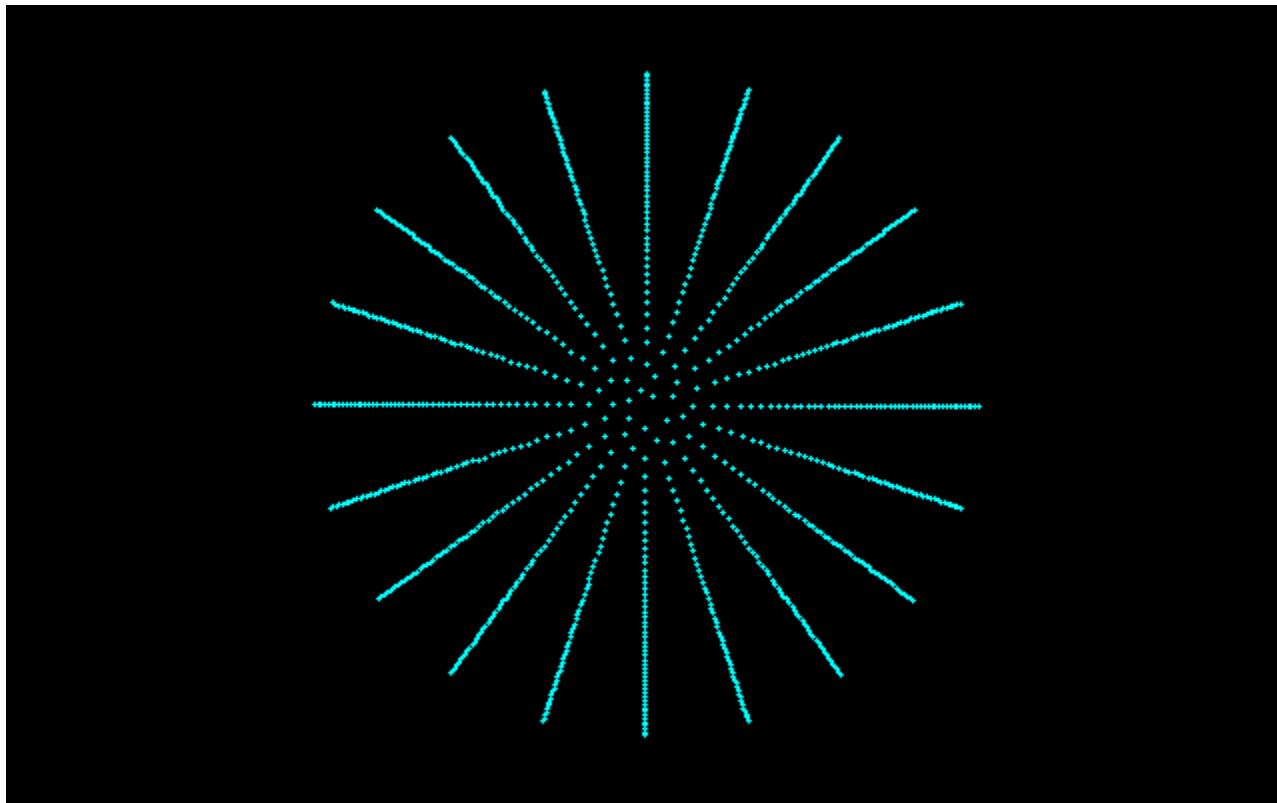


222.9

# Margaret (21=8+13)

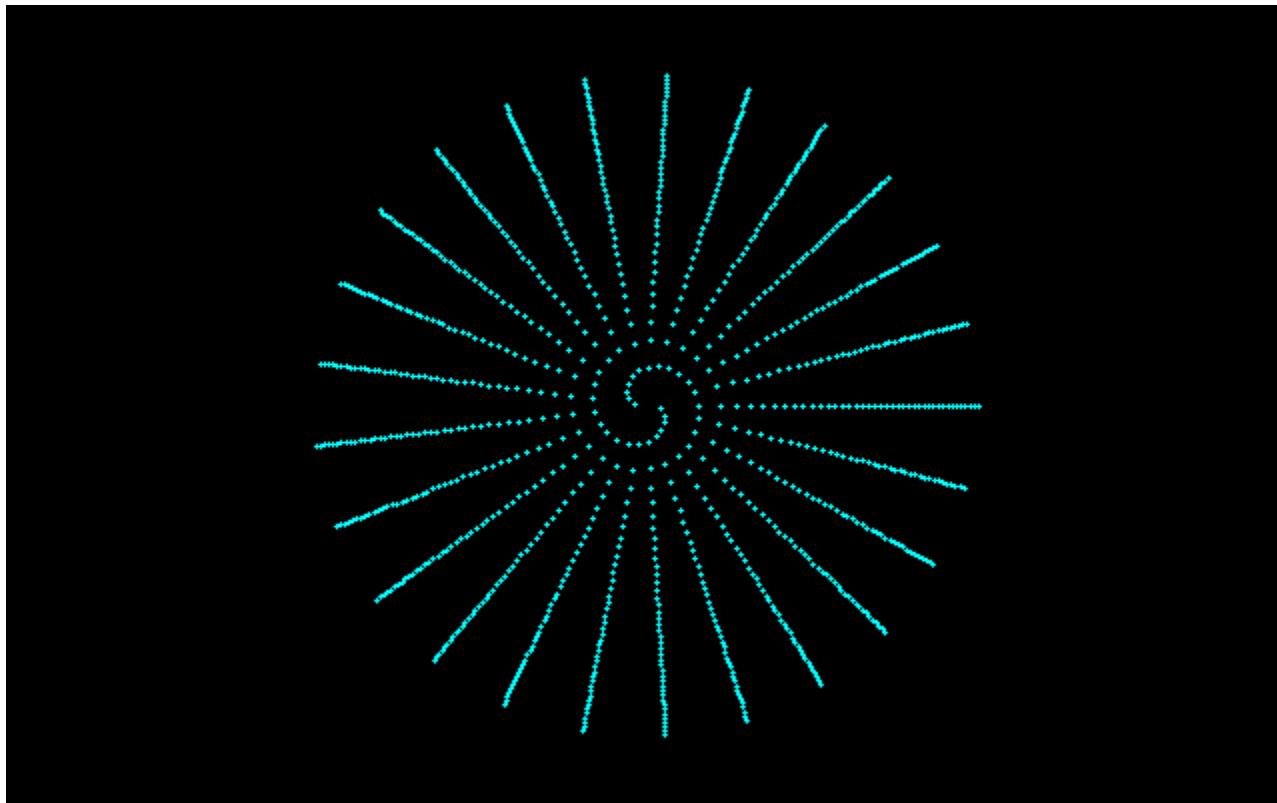


$$\alpha = 3/20 = [0; 6, 1, 2]$$



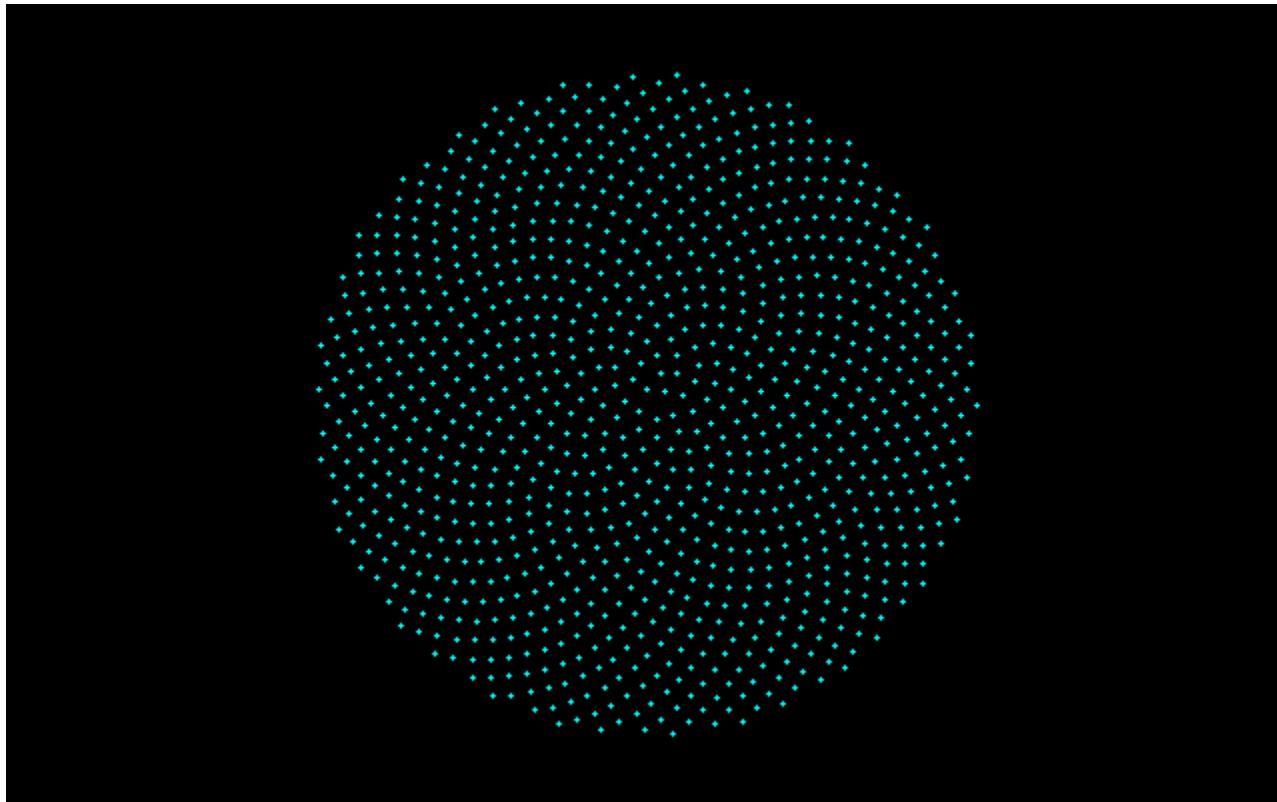
54.0

$$\alpha = 12/25 = [0; 2, 12]$$



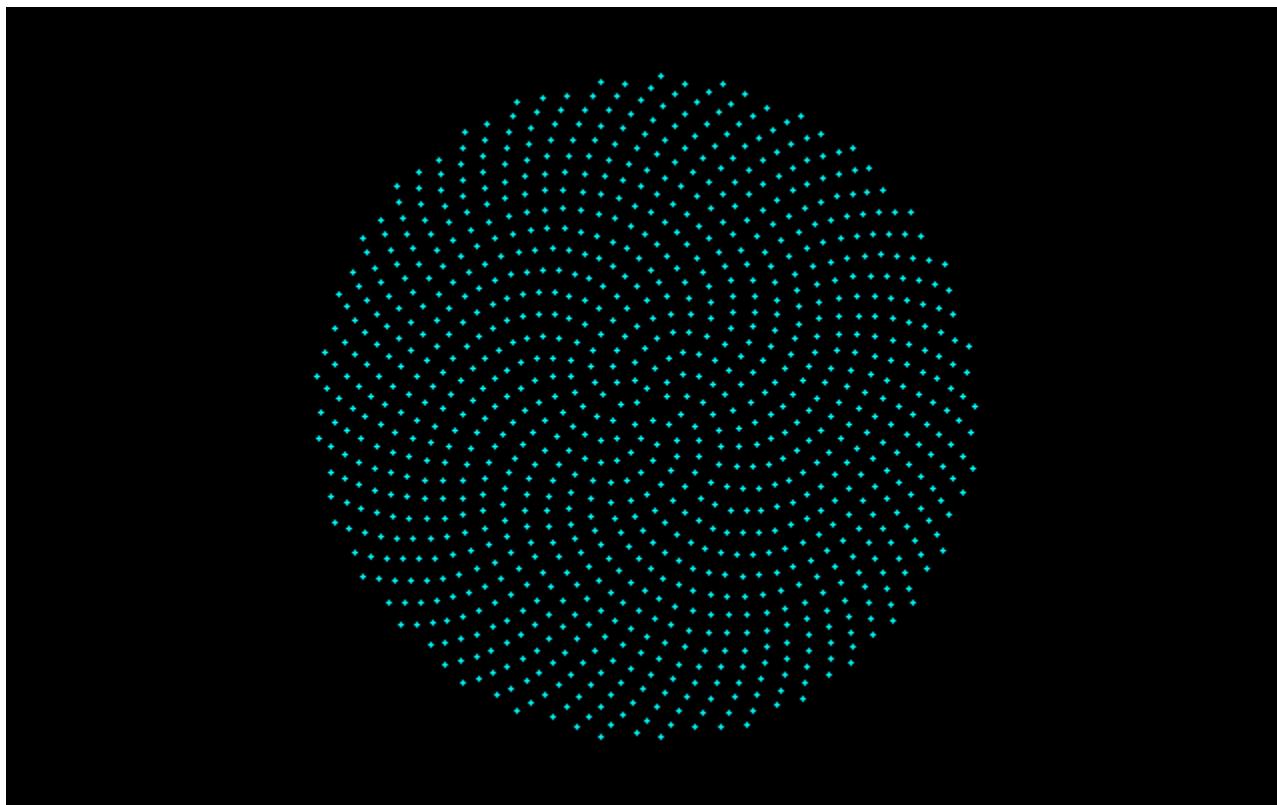
172.8

$$1/\varphi = (\sqrt{5}-1)/2 = [0;1,1,1,1,1,1,\dots] = 0.618$$



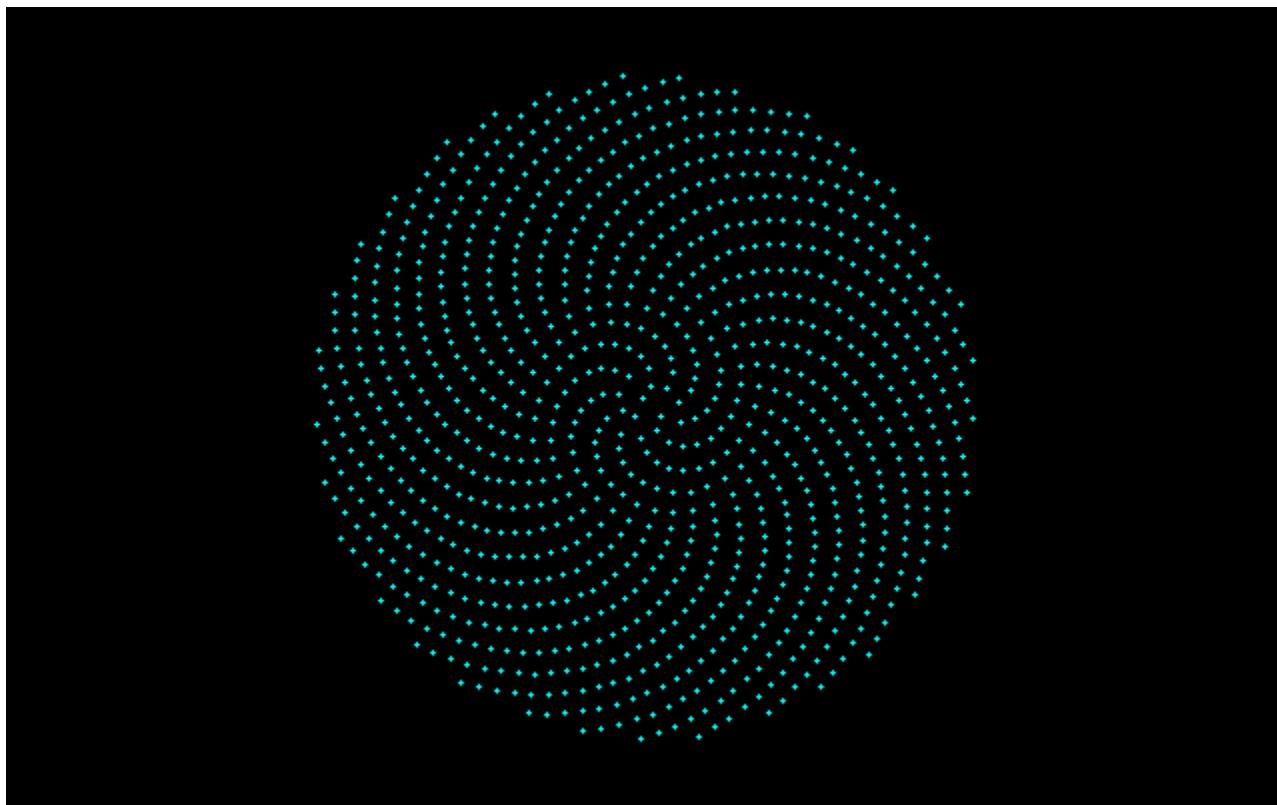
222.5

$$\alpha = \sqrt{2} - 1 = [0; 2, 2, 2, 2, \dots] = 0.414$$



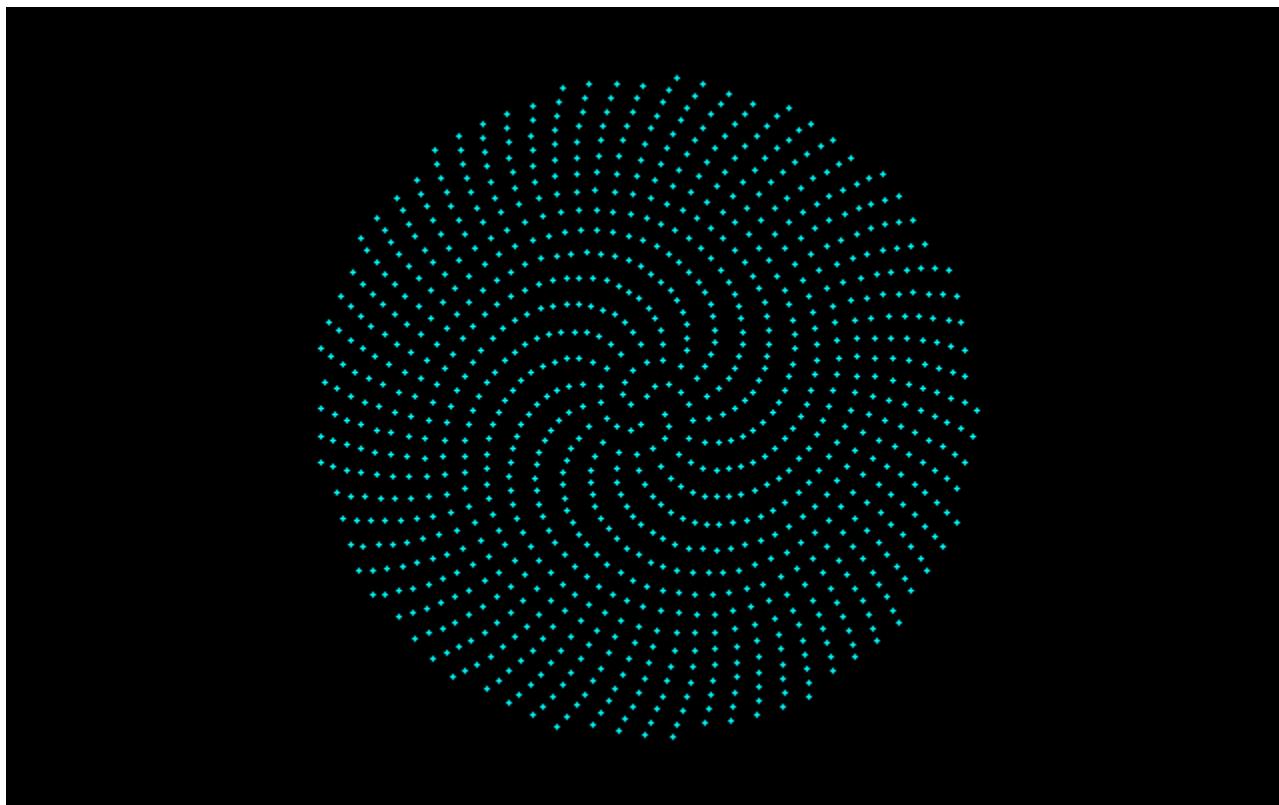
149.1

$$\alpha = (\sqrt{13} - 3)/2 = [0; 3, 3, 3, 3, \dots]$$



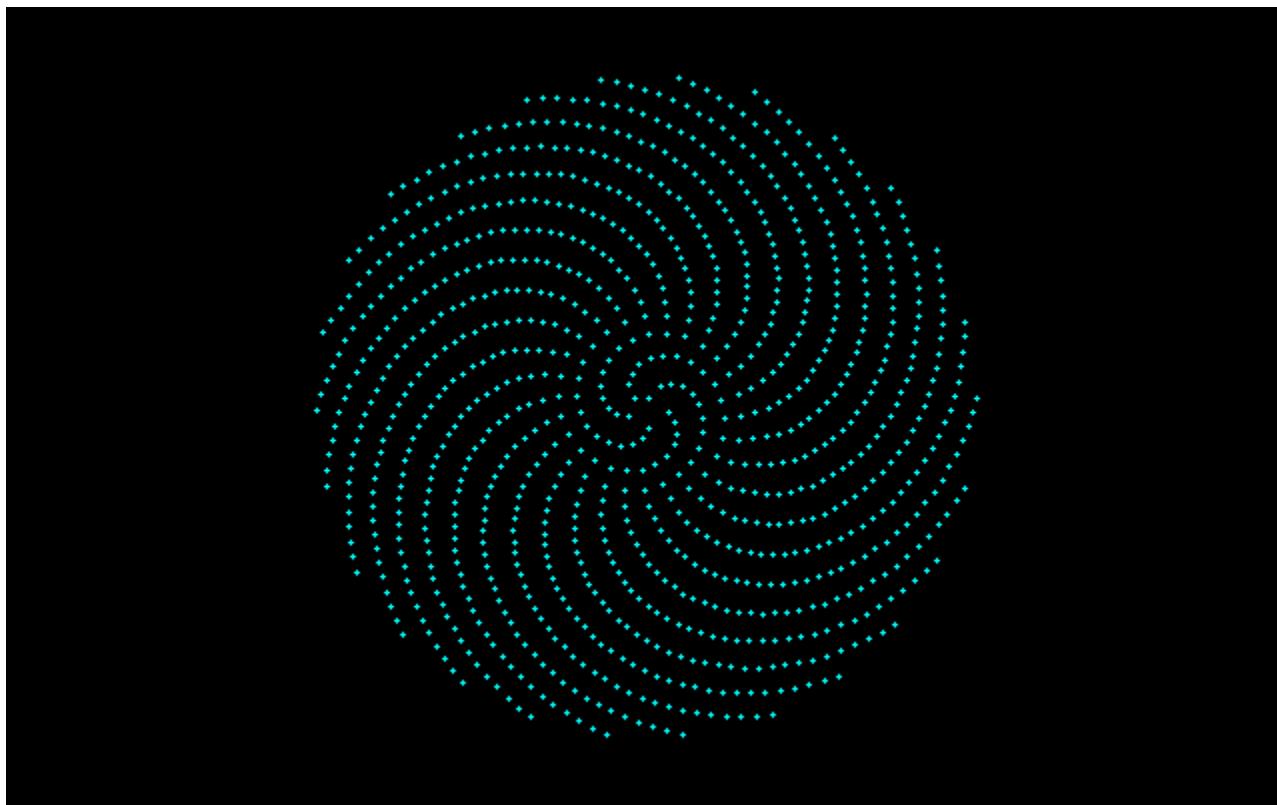
109.0

$$\alpha = \sqrt{5} - 2 = [0; 4, 4, 4, 4, \dots]$$



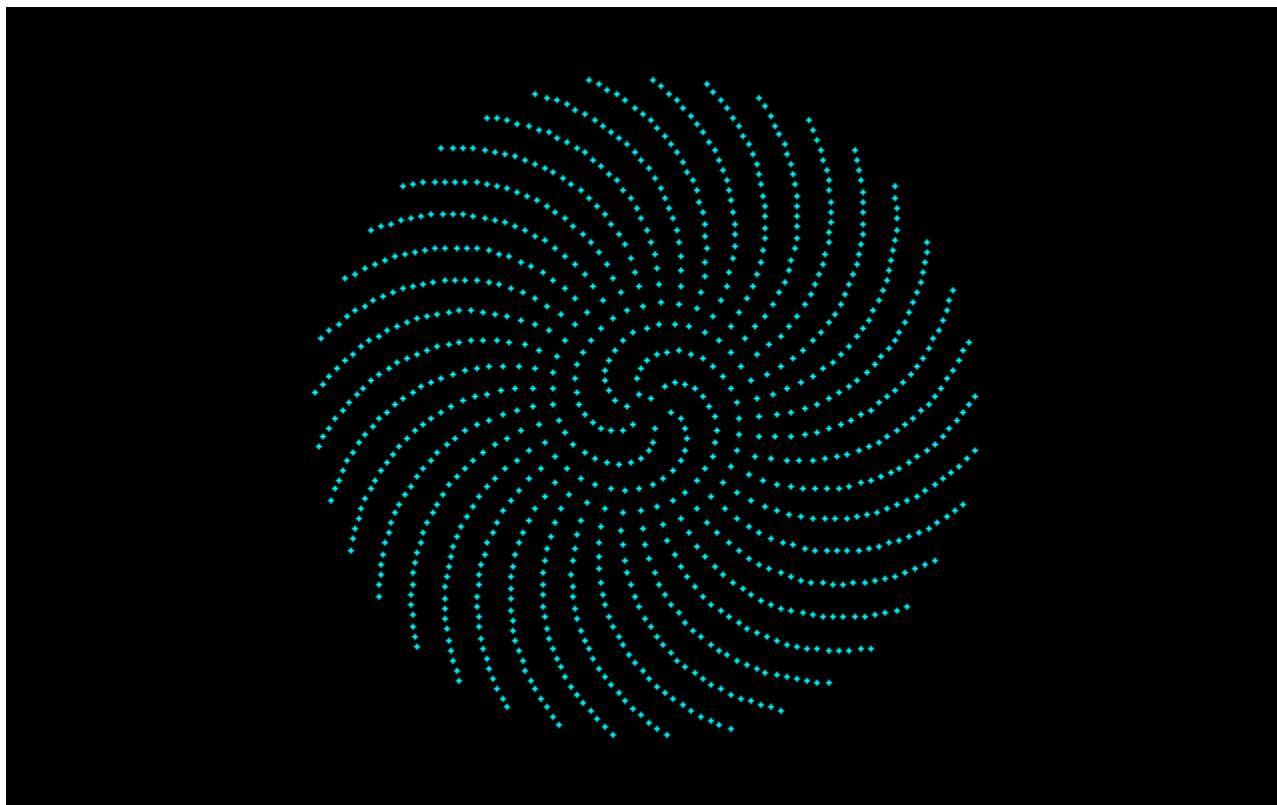
85.0

$$\alpha = (\sqrt{29} - 5)/2 = [0; 5, 5, 5, 5, \dots]$$



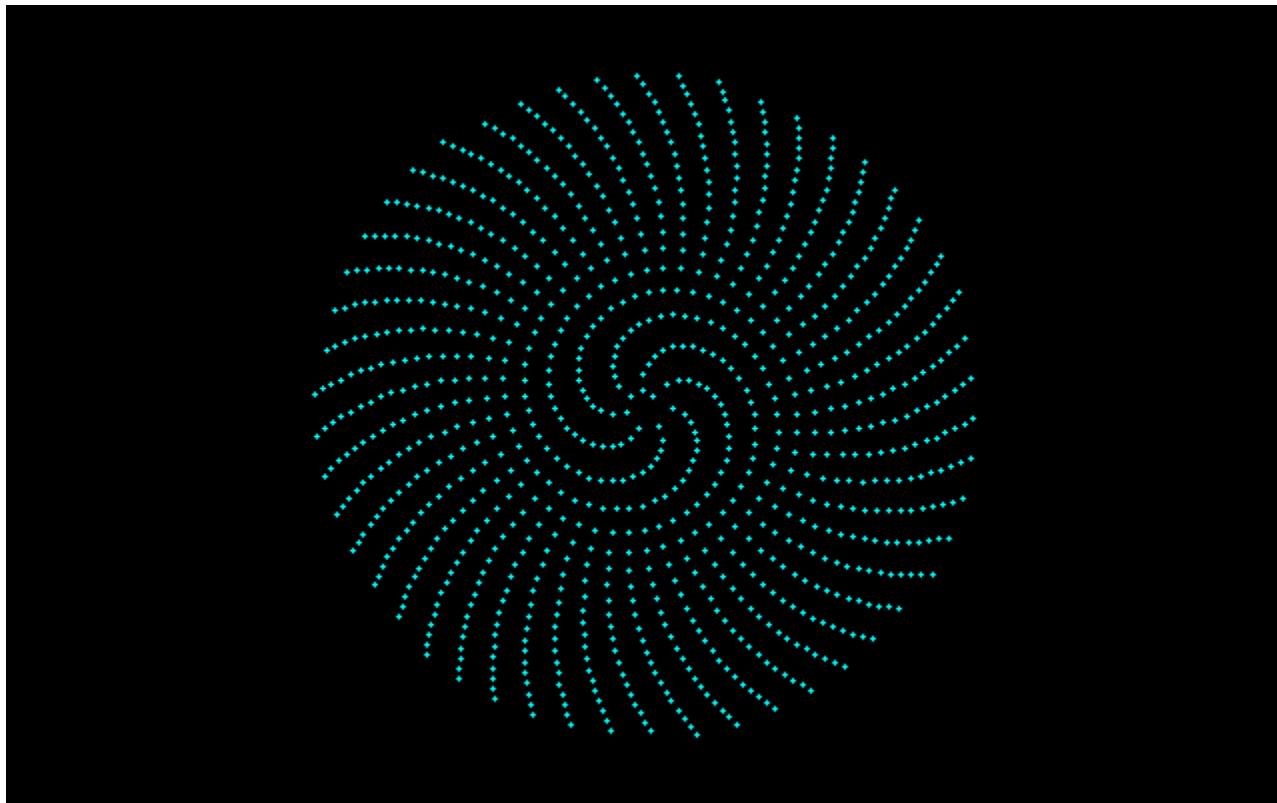
69.3

$$\alpha = \sqrt{10} - 3 = [0; 6, 6, 6, 6, \dots]$$



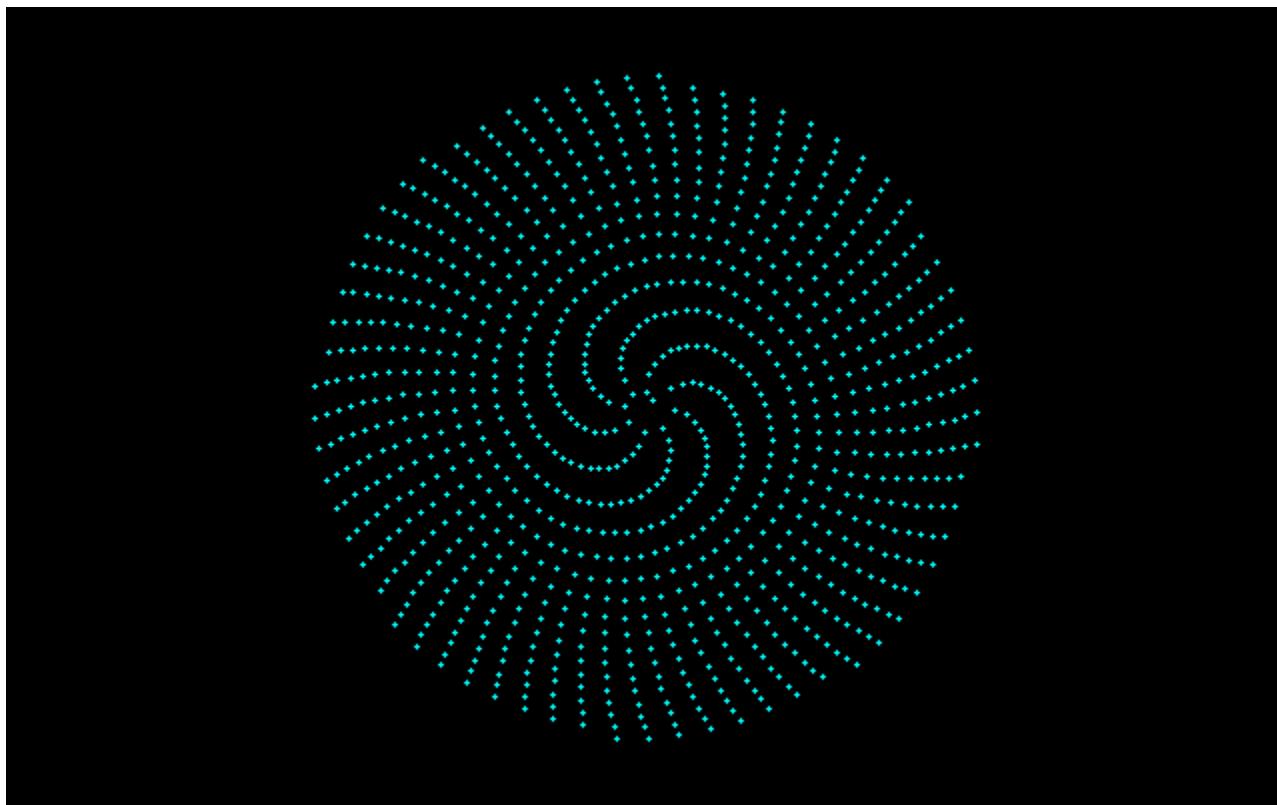
58.4

$$\alpha = (\sqrt{53} - 7)/2 = [0; 7, 7, 7, 7, \dots]$$



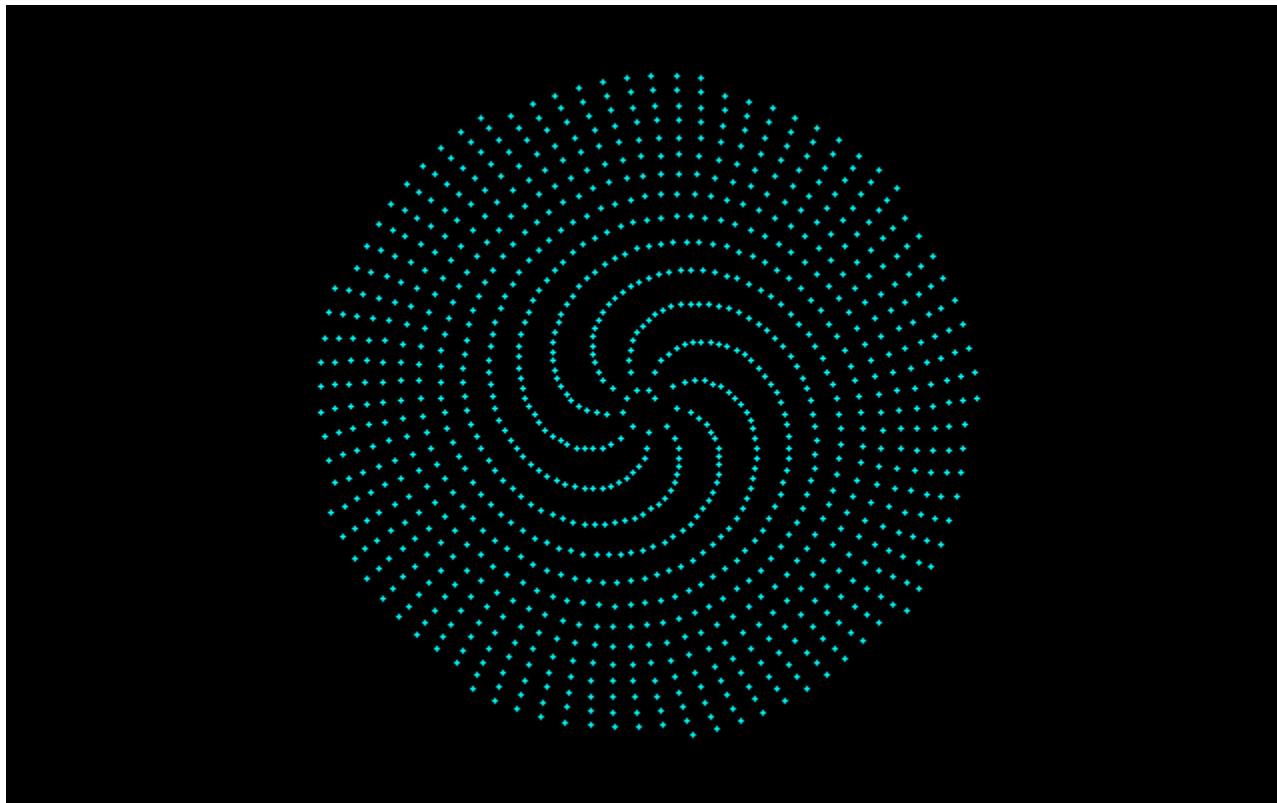
50.4

$$\alpha = \sqrt{17} - 4 = [0; 8, 8, 8, 8, \dots]$$



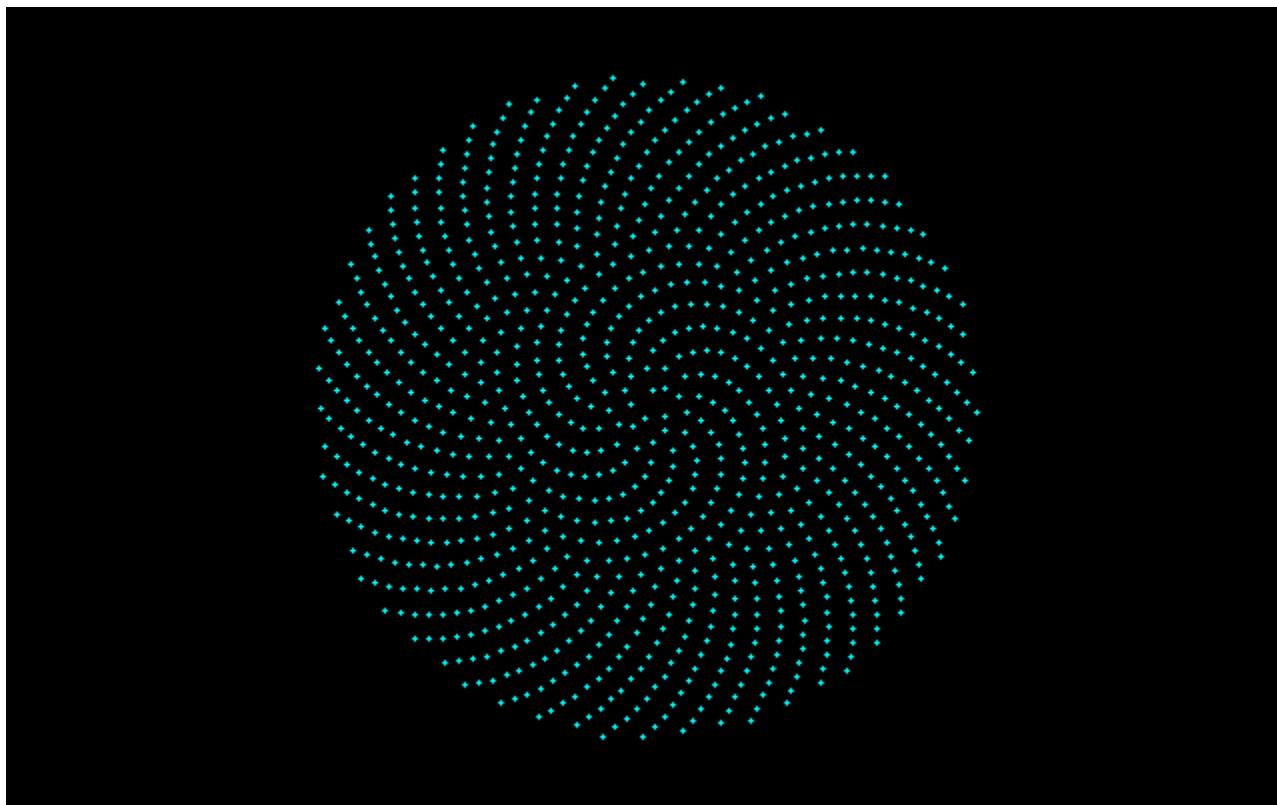
44.3

$$\alpha = (\sqrt{85} - 9)/2 = [0; 9, 9, 9, 9, \dots]$$



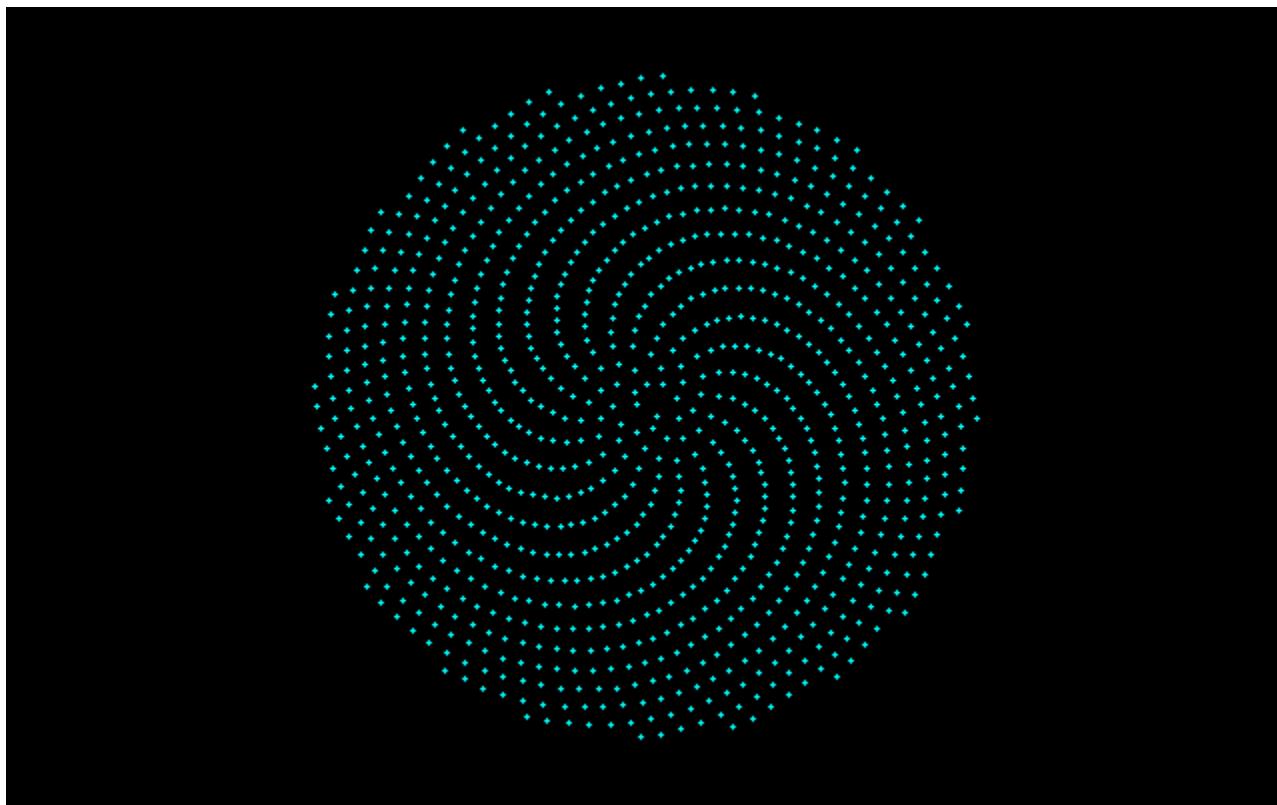
39.5

$$\alpha = \sqrt{3} - 1 = [0; 1, 2, 1, 2, \dots]$$



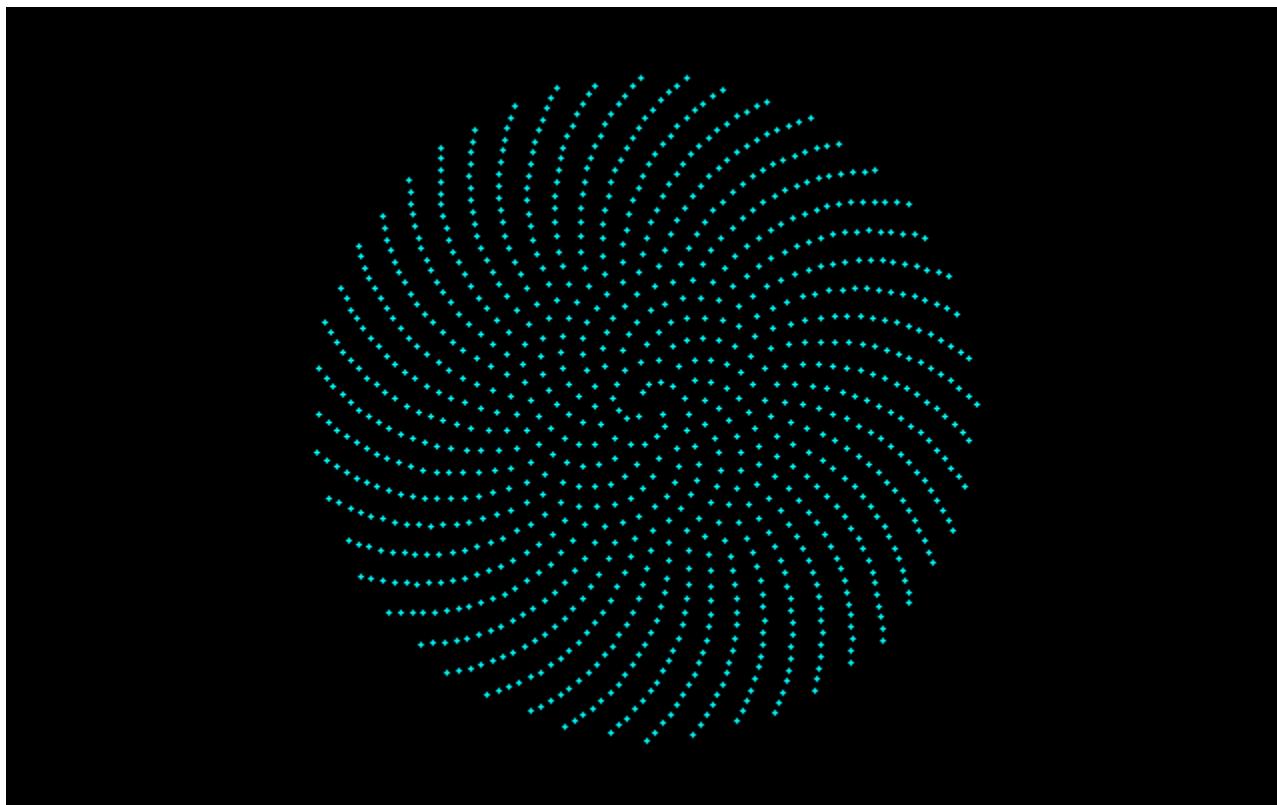
263.5

$$\alpha = \sqrt{6} - 2 = [0; 2, 4, 2, 4, \dots]$$



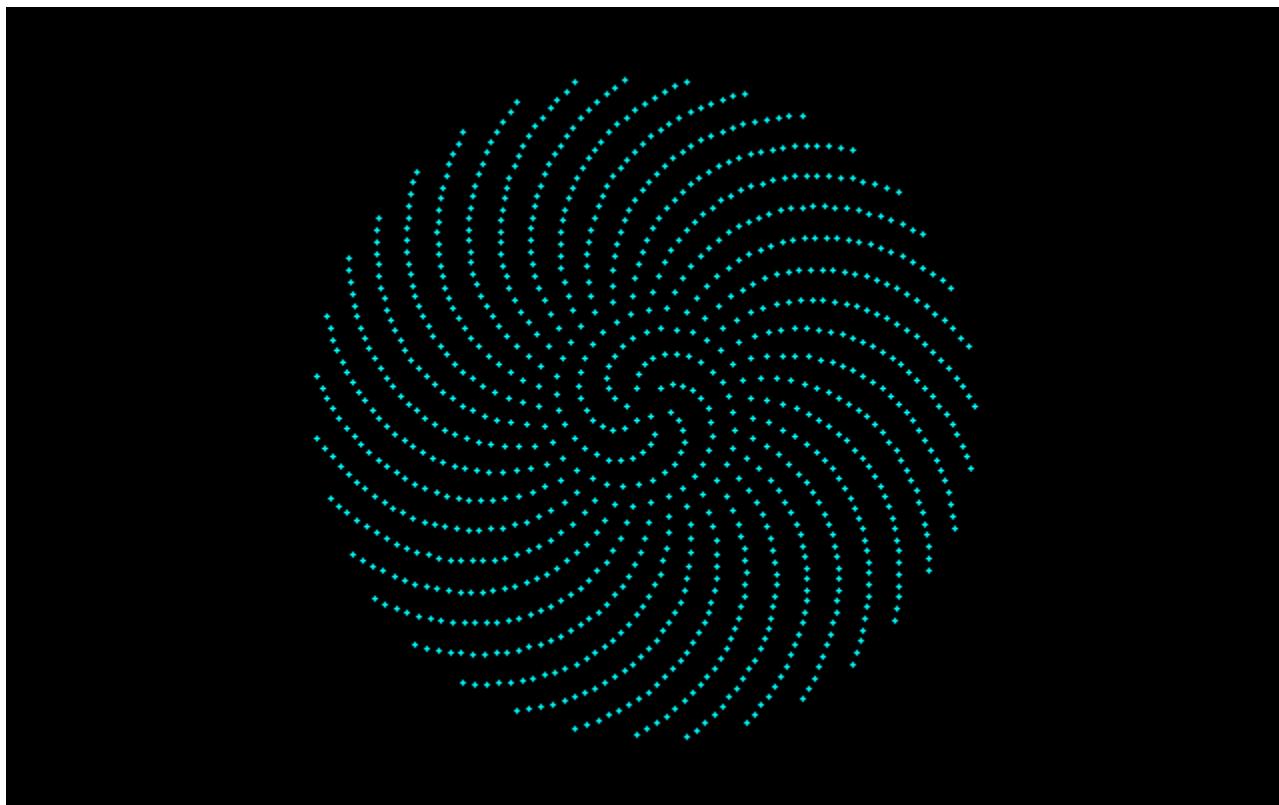
161.8

$$\alpha = \sqrt{7} - 2 = [0; 1, 1, 1, 4, 1, 1, 1, 4, \dots]$$



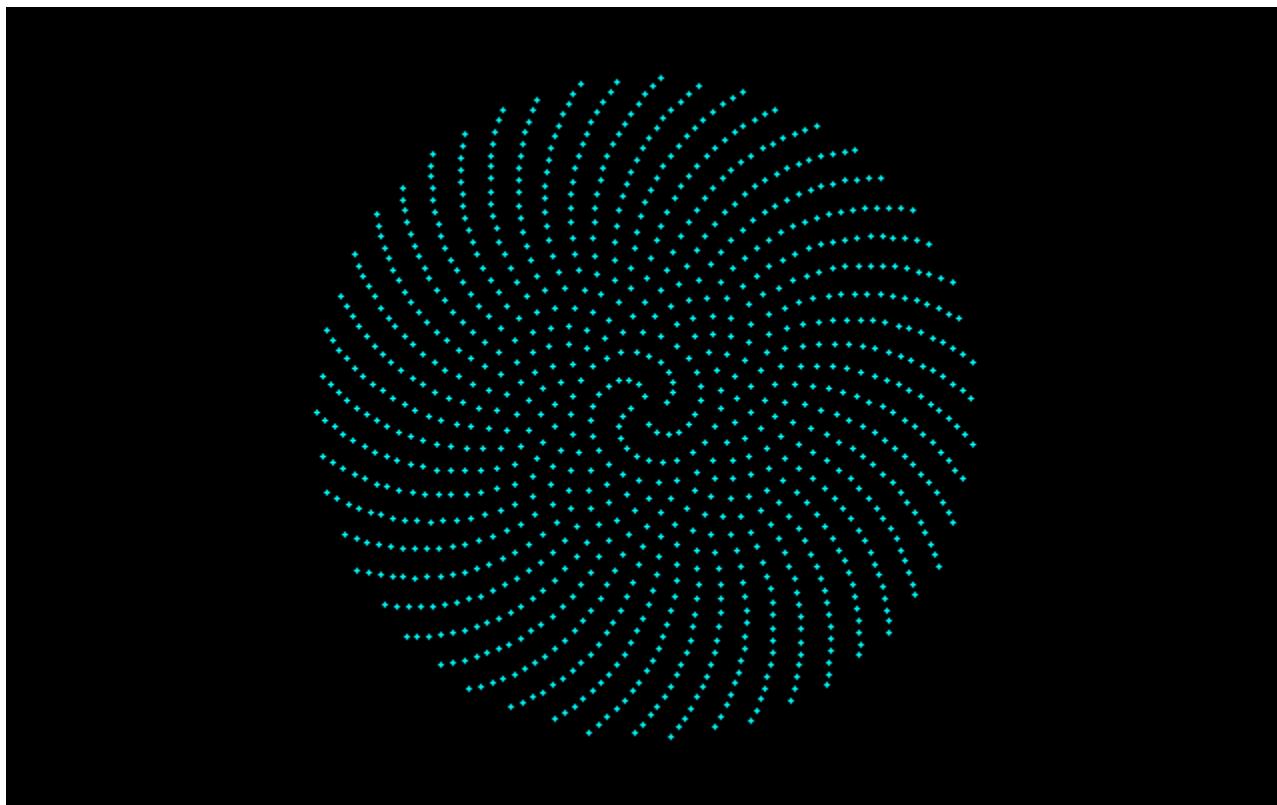
232.5

$$\alpha = \sqrt{8} - 2 = [0; 1, 4, 1, 4, \dots]$$



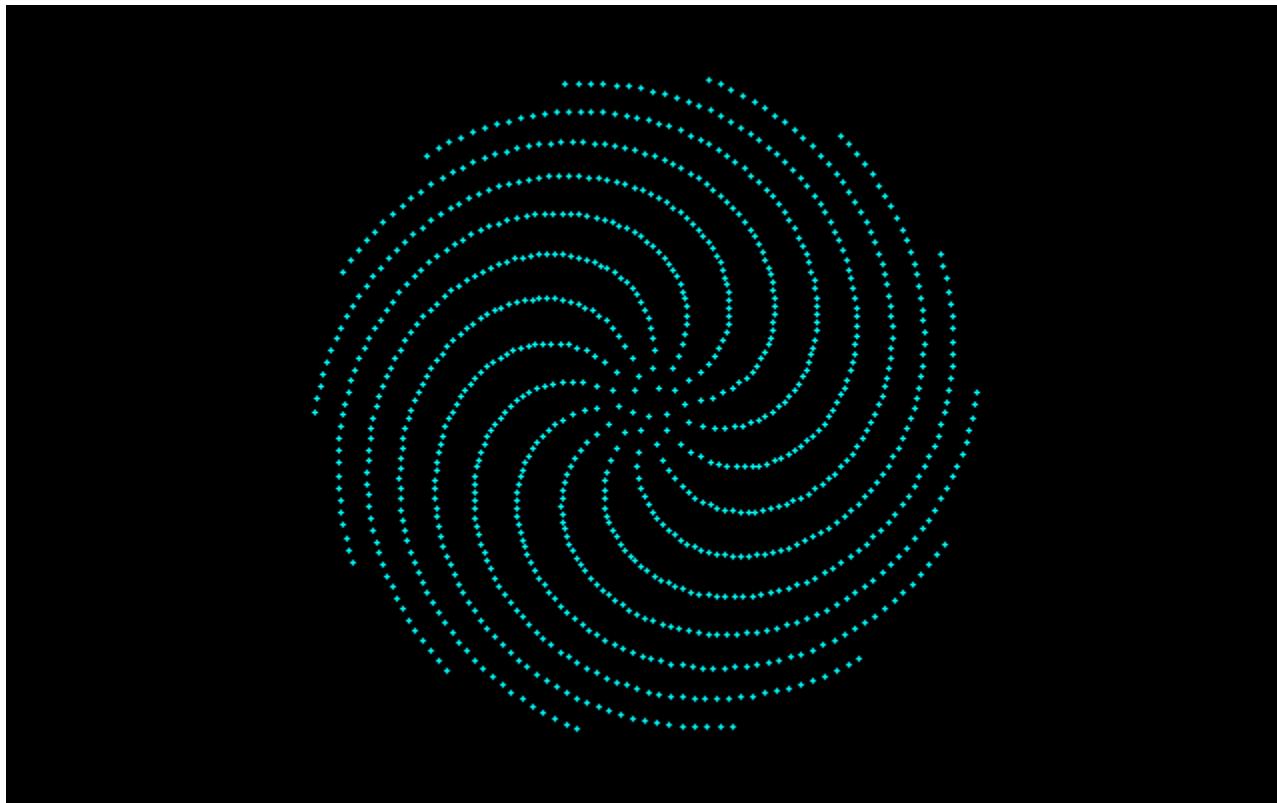
298.2

$$\sqrt[3]{2}-1 = [0;3,1,5,1,1,4,1,1,\dots]$$



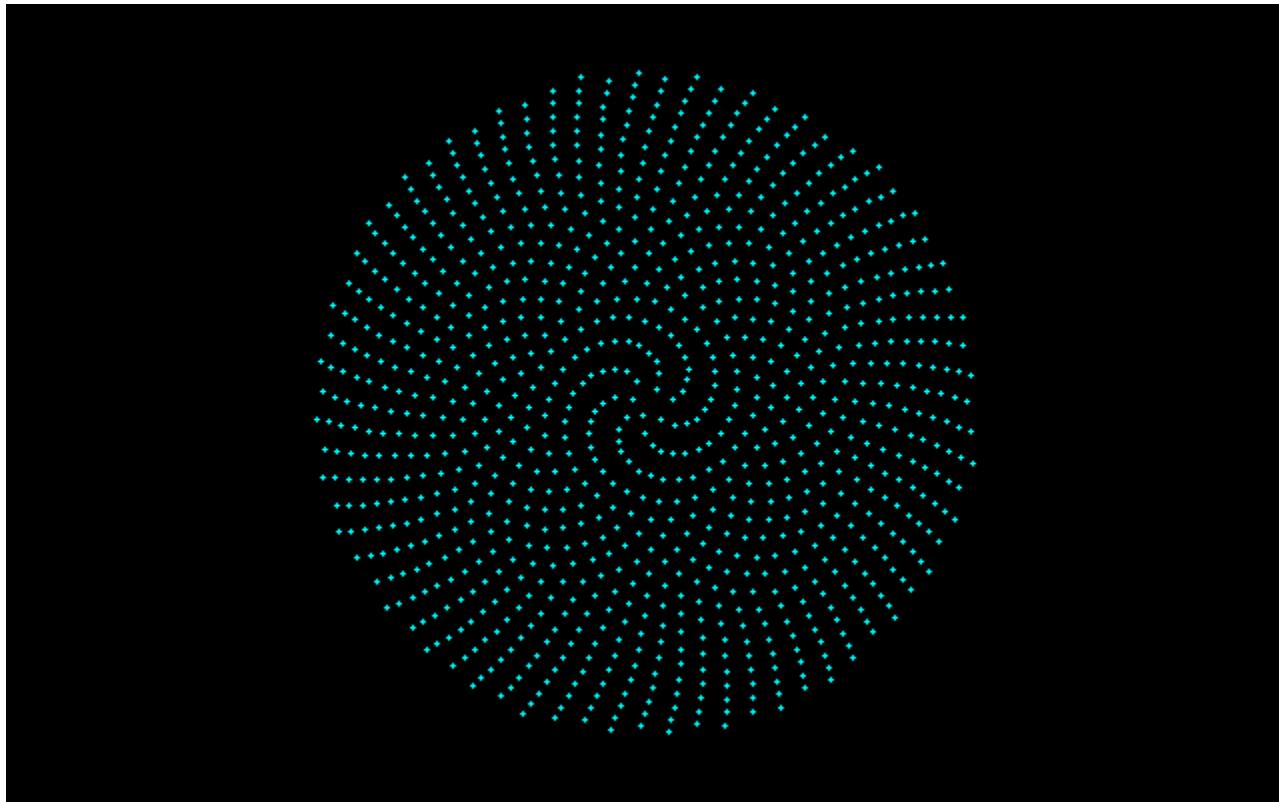
93.6

$$1/\varphi^{1/2} = [0; 1, 3, 1, 2, 13, 3, 5, 1, \dots]$$



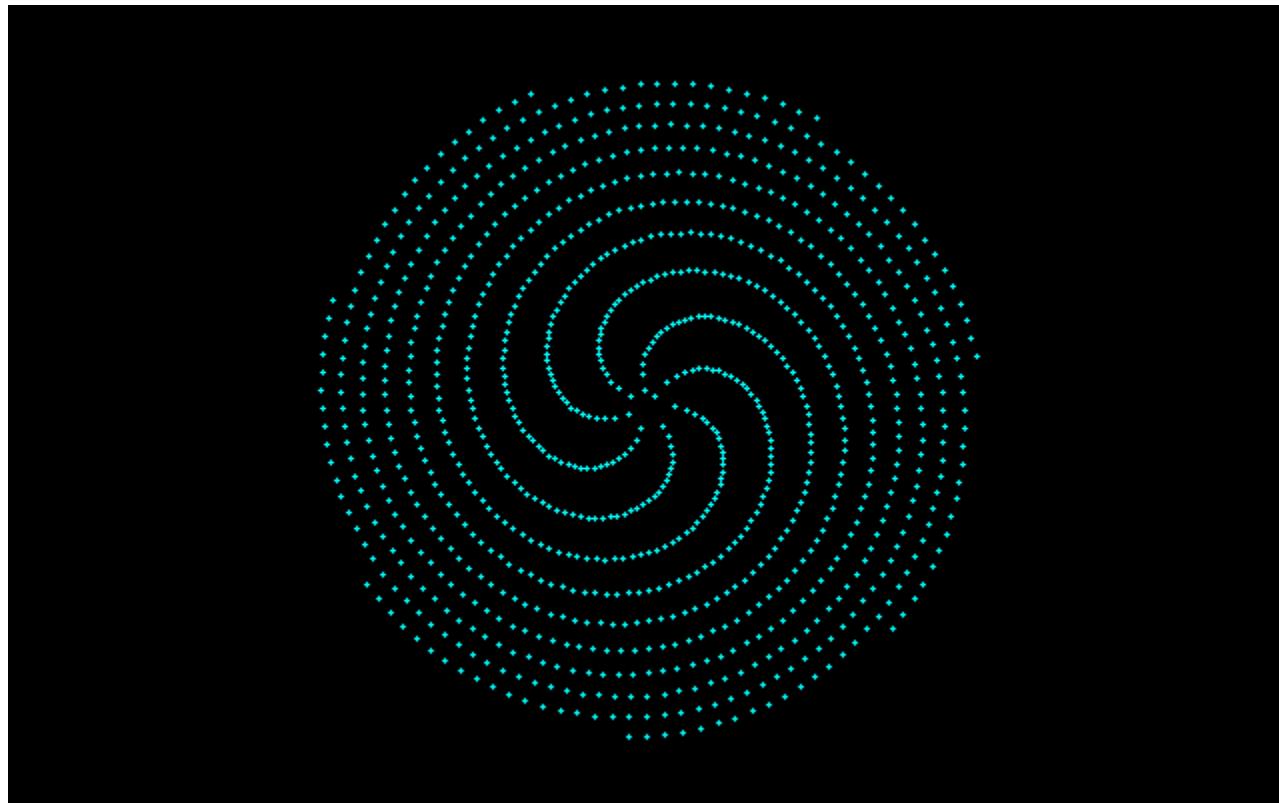
283.0

$$e-2 = [0; 1, 2, 1, 1, 4, 1, 1, 6, \dots] = 0.718$$



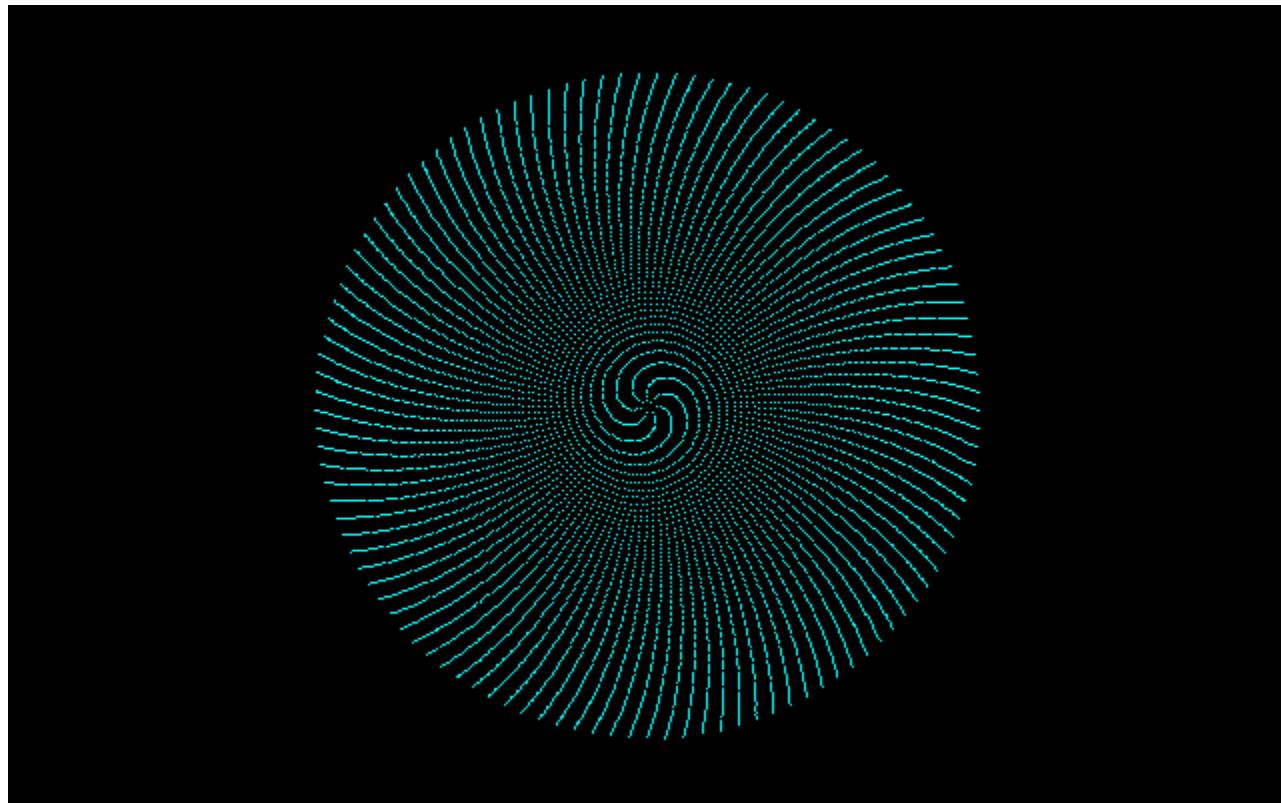
258.6

$$\pi - 3 = [0; 7, 15, 1, 292, 1, 1, 1, 2, \dots] = 0.142$$



51.0

$$\pi - 3 = [0; 7, 15, 1, 292, 1, 1, 1, 2, \dots] = 0.142$$



51.0

# Diophantine approximation

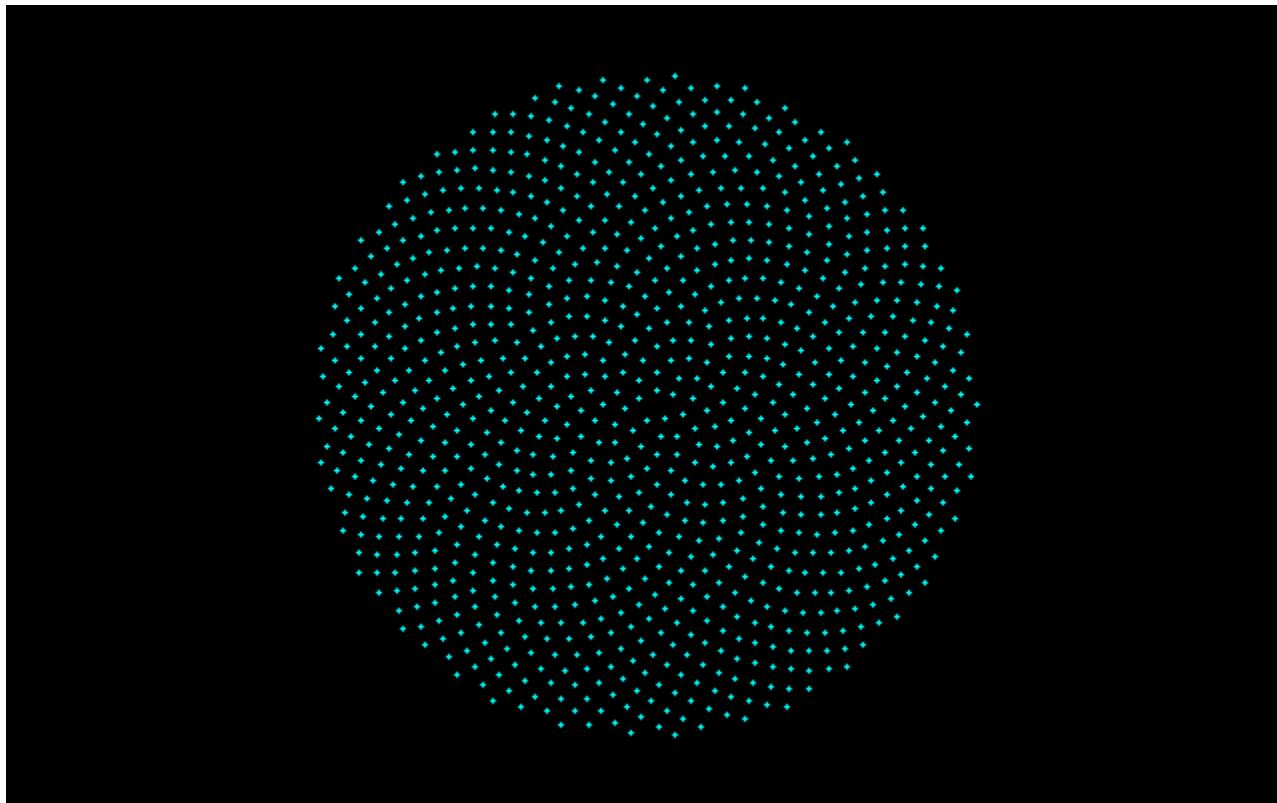
- $|\pi - 3/1|$                        $3/1 = [3;]$
- $|\pi - 22/7|$                        $22/7 = [3;7]$
- $|\pi - 333/106|$                  $333/106 = [3;7,15]$
- $|\pi - 355/113|$                  $355/113 = [3;7,15,1]$
- $|\pi - 103993/33102|$
- $|\pi - 104348/33215|$
- $|\pi - 208341/66317|$
- $|\pi - 312689/99532|$
- $\pi = [3;7,15,1,292,1,1,1,2, \dots]$

$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}$$

$\alpha$ の連分数展開の部分商が  
1個を除きすべて1の場合

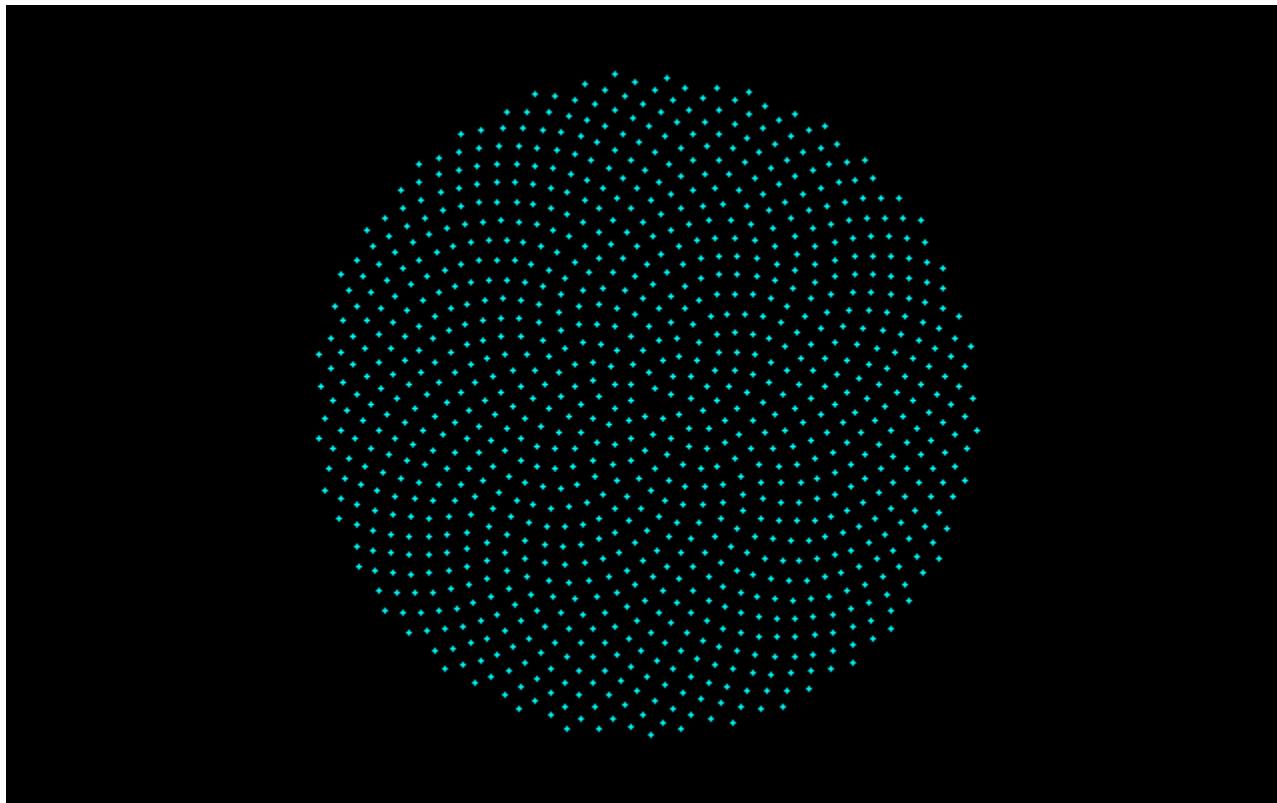
フィボナッチパターンが現れるのは  
黄金角のときには限る？

黃金角,  $\alpha=1/(\varphi+1)$   
=[0;2,1,1,1,1,1,1,1,1,· · ·]



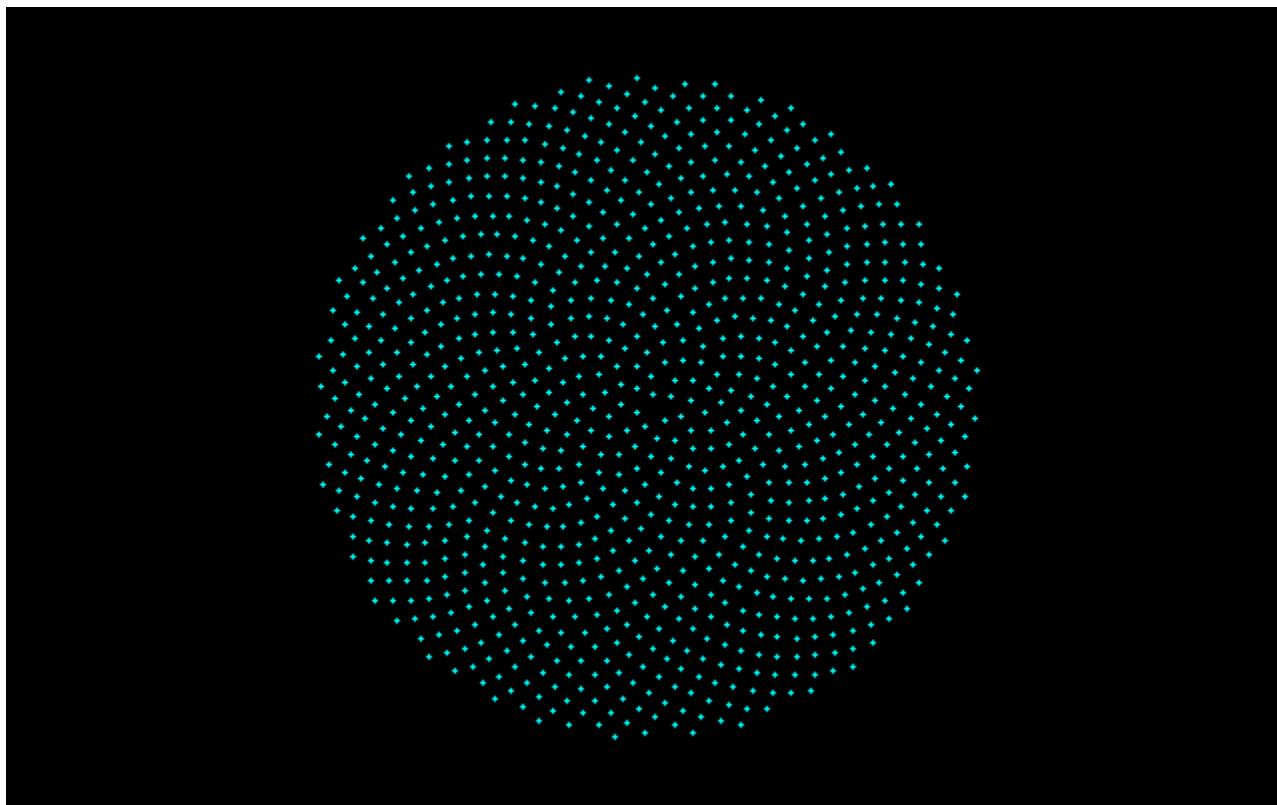
137.5

$$\alpha = (\varphi + 1) / (\varphi + 2)$$
$$= [0; 1, 2, 1, 1, 1, 1, 1, 1, \dots]$$



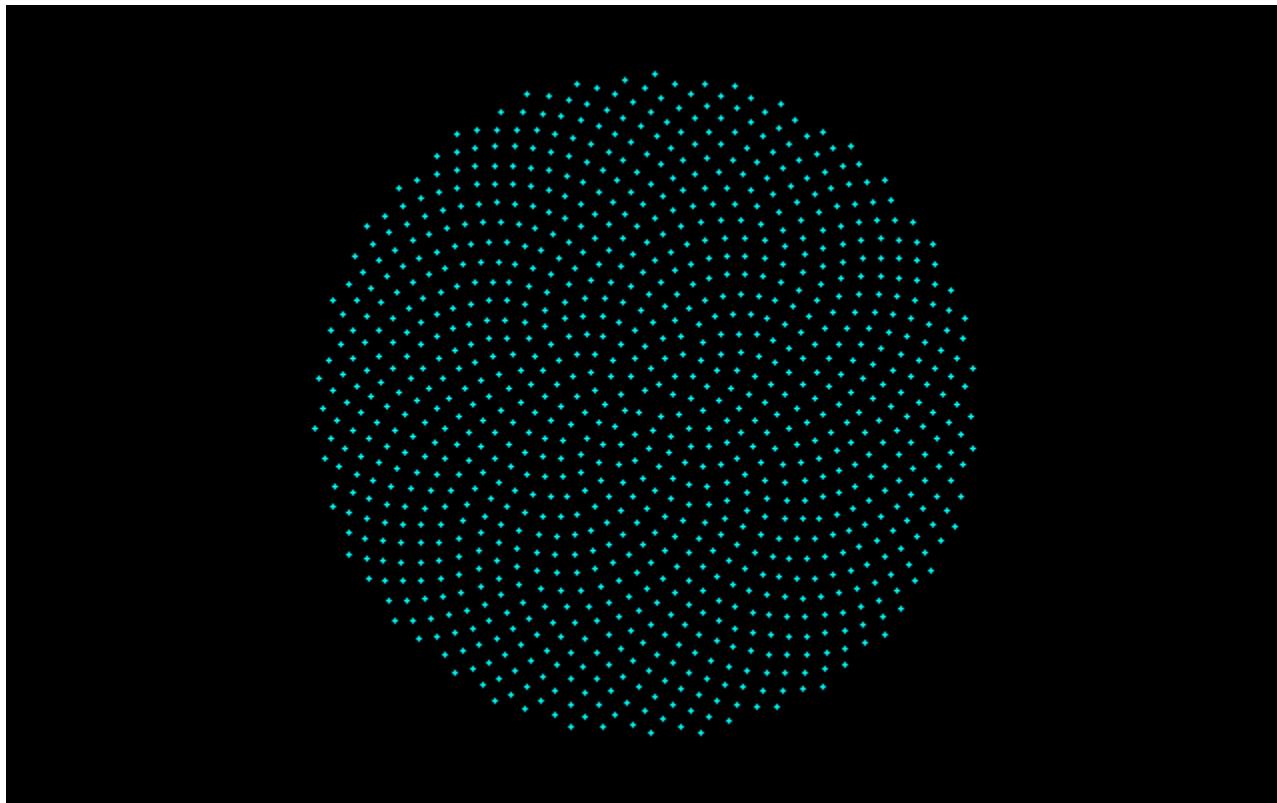
260.5

$$\begin{aligned}\alpha &= (\varphi+2)/(2\varphi+3) \\ &=[0;1,1,2,1,1,1,1,1,\dots]\end{aligned}$$



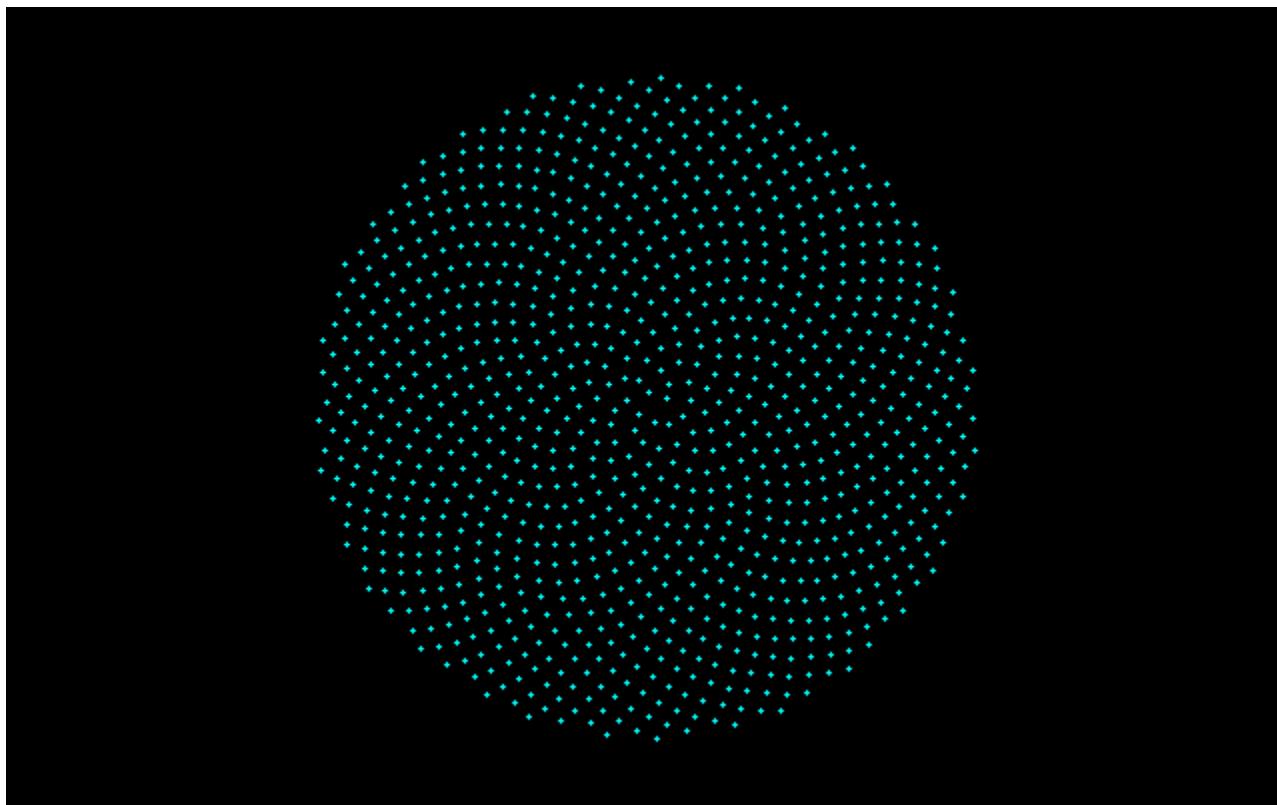
208.9

$$\alpha = \frac{2\varphi + 3}{3\varphi + 5}$$
$$= [0; 1, 1, 1, 2, 1, 1, 1, 1, \dots]$$



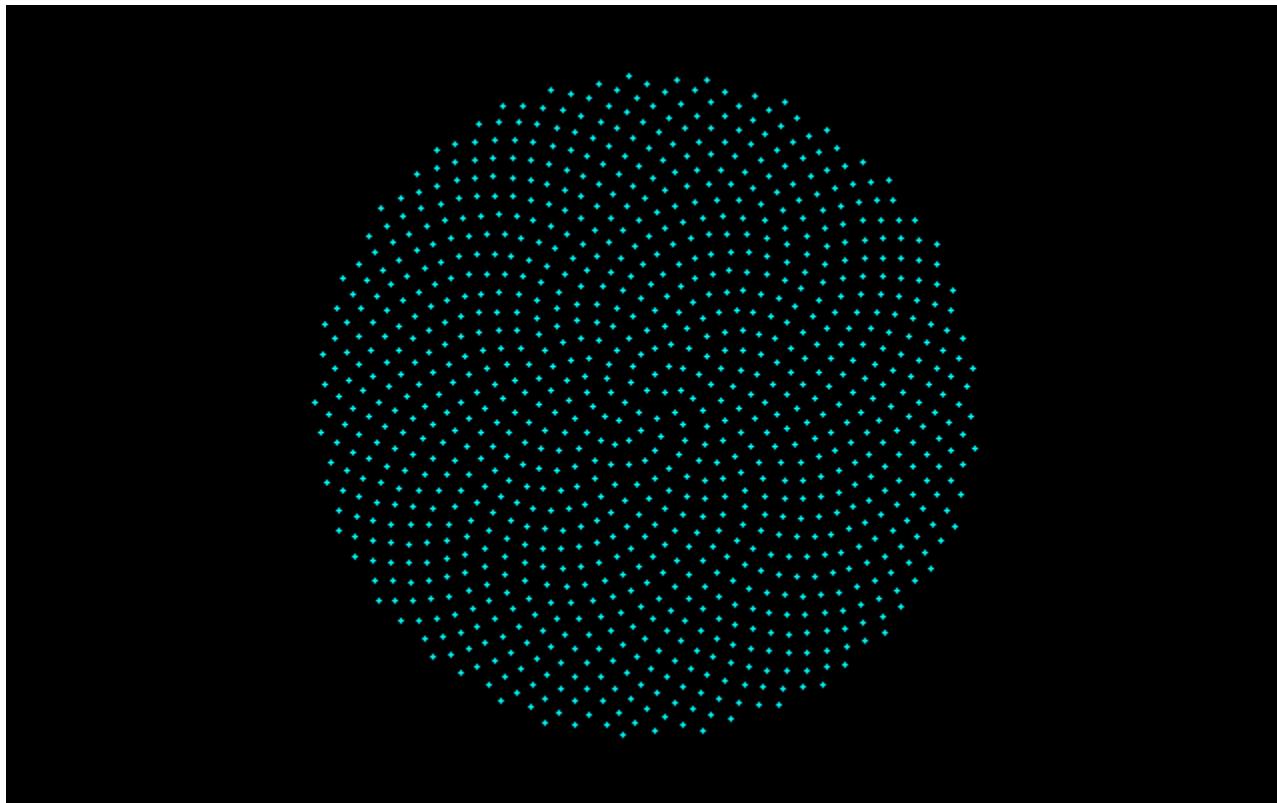
227.8

$$\begin{aligned}\alpha &= (3\varphi + 5)/(5\varphi + 8) \\ &= [0; 1, 1, 1, 1, 2, 1, 1, 1, \dots]\end{aligned}$$



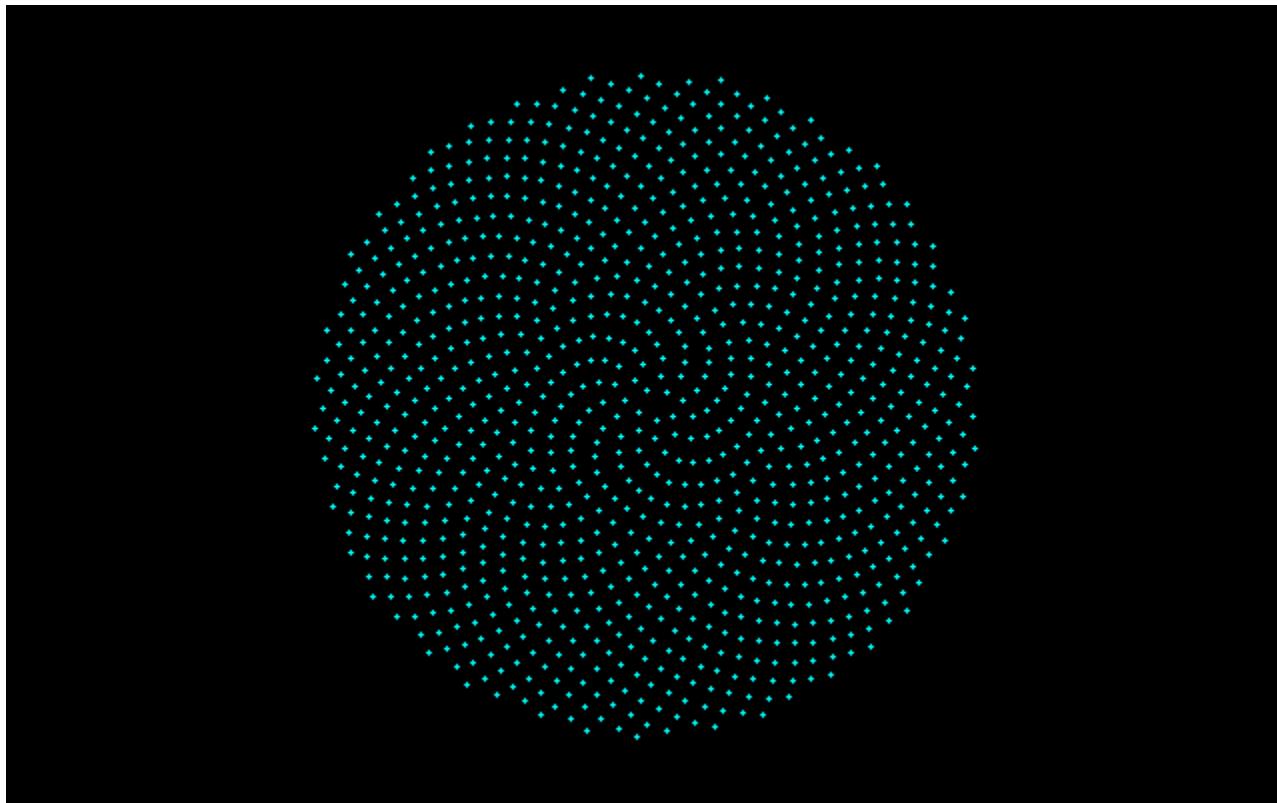
220.5

$$\begin{aligned} \alpha &= (5\varphi + 8)/(8\varphi + 13) \\ &= [0; 1, 1, 1, 1, 1, 2, 1, 1, \dots] \end{aligned}$$



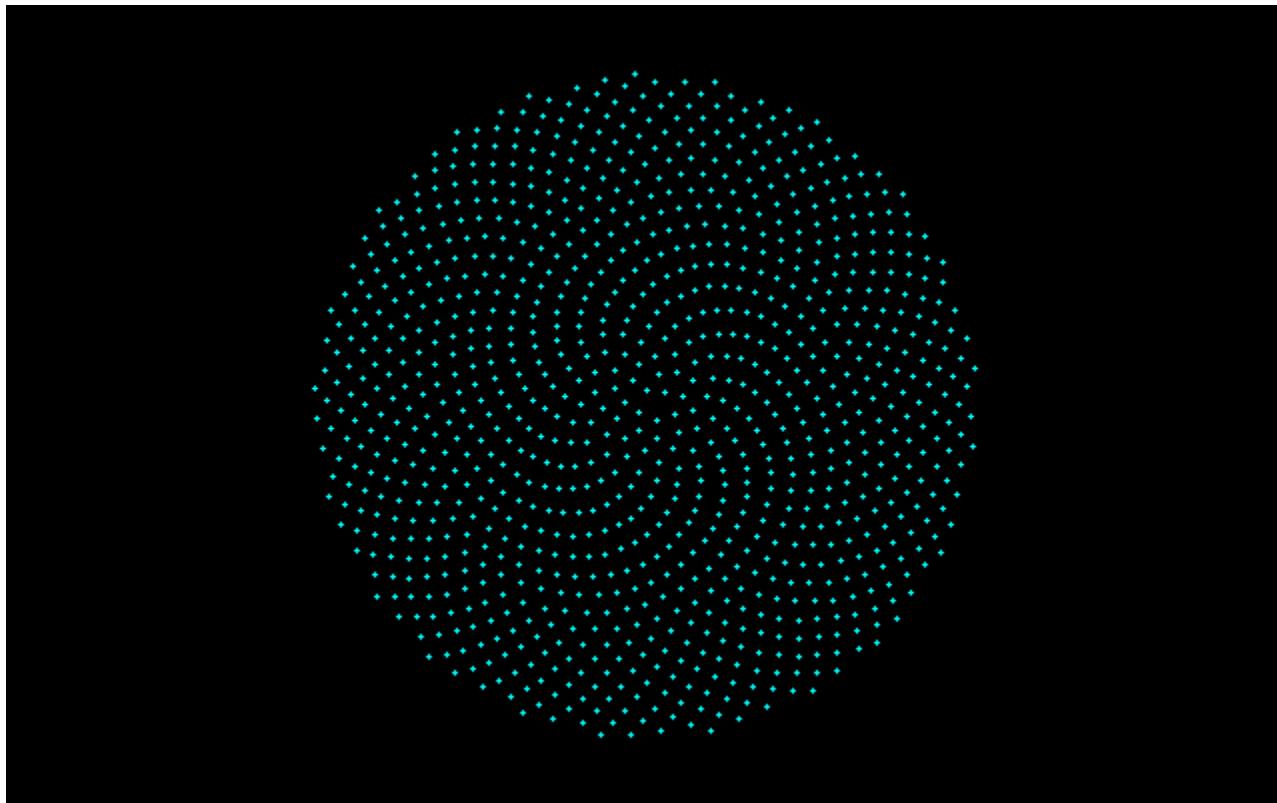
223.3

$$\begin{aligned}\alpha &= (8\varphi + 13)/(13\varphi + 21) \\ &= [0; 1, 1, 1, 1, 1, 1, 2, 1, \dots]\end{aligned}$$



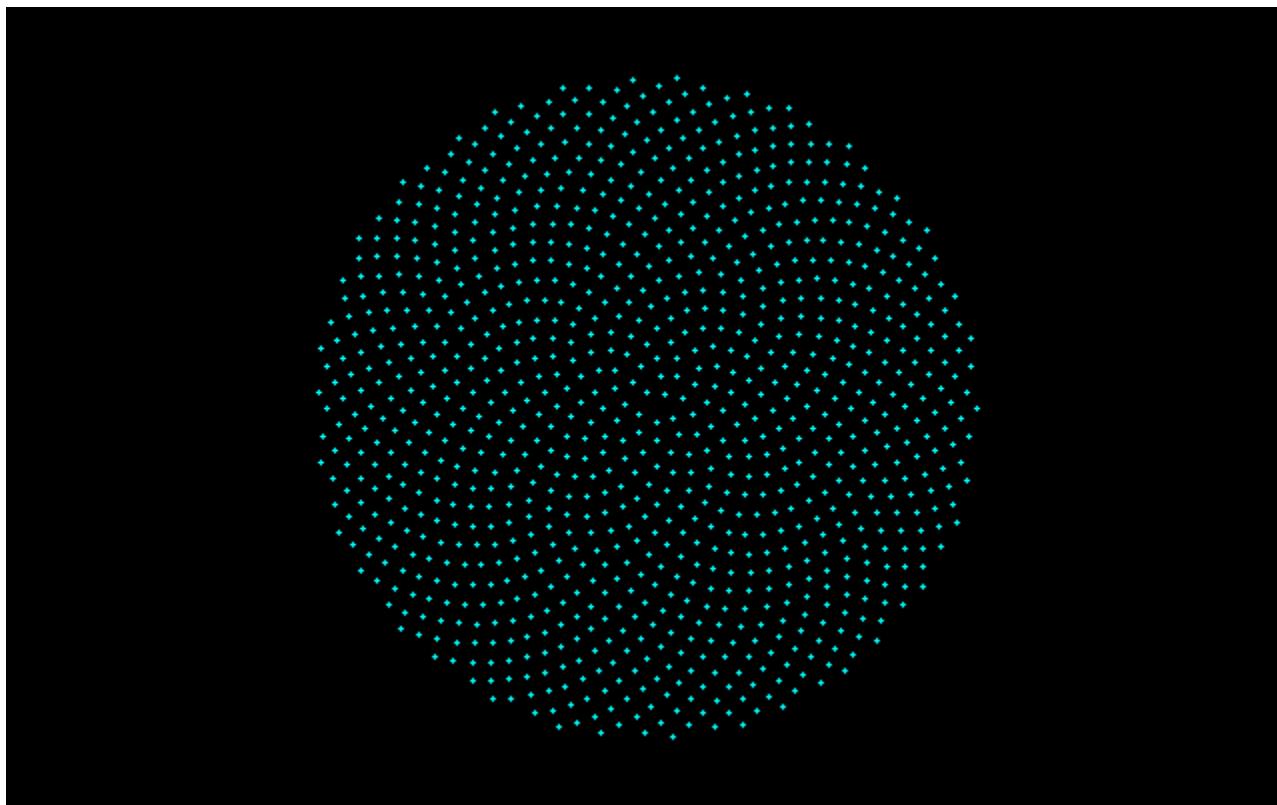
222.2

$$\begin{aligned} a &= (13\varphi + 21)/(21\varphi + 34) \\ &= [0; 1, 1, 1, 1, 1, 1, 1, 2, \dots] \end{aligned}$$



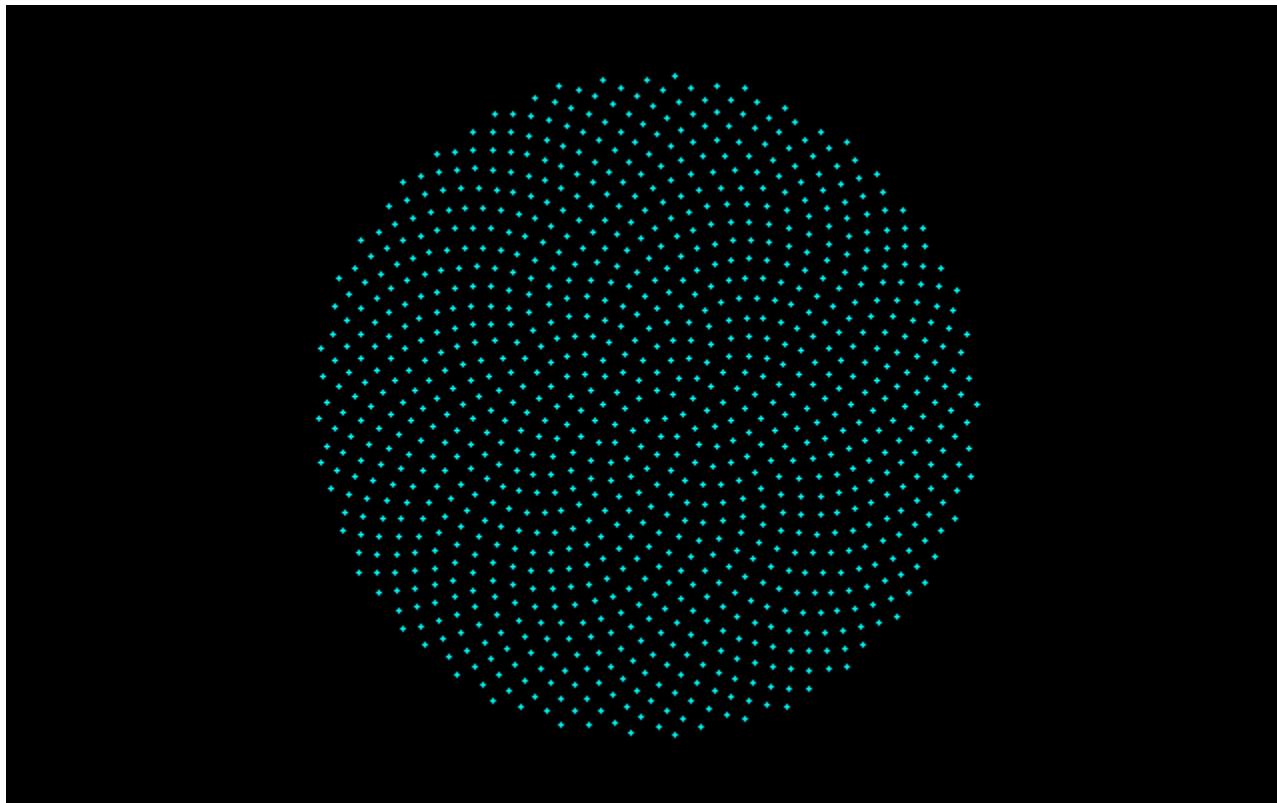
222.6

$$\alpha = 1/(\varphi + 0)$$
$$=[0; 1, 1, 1, 1, 1, 1, 1, 1, \dots]$$



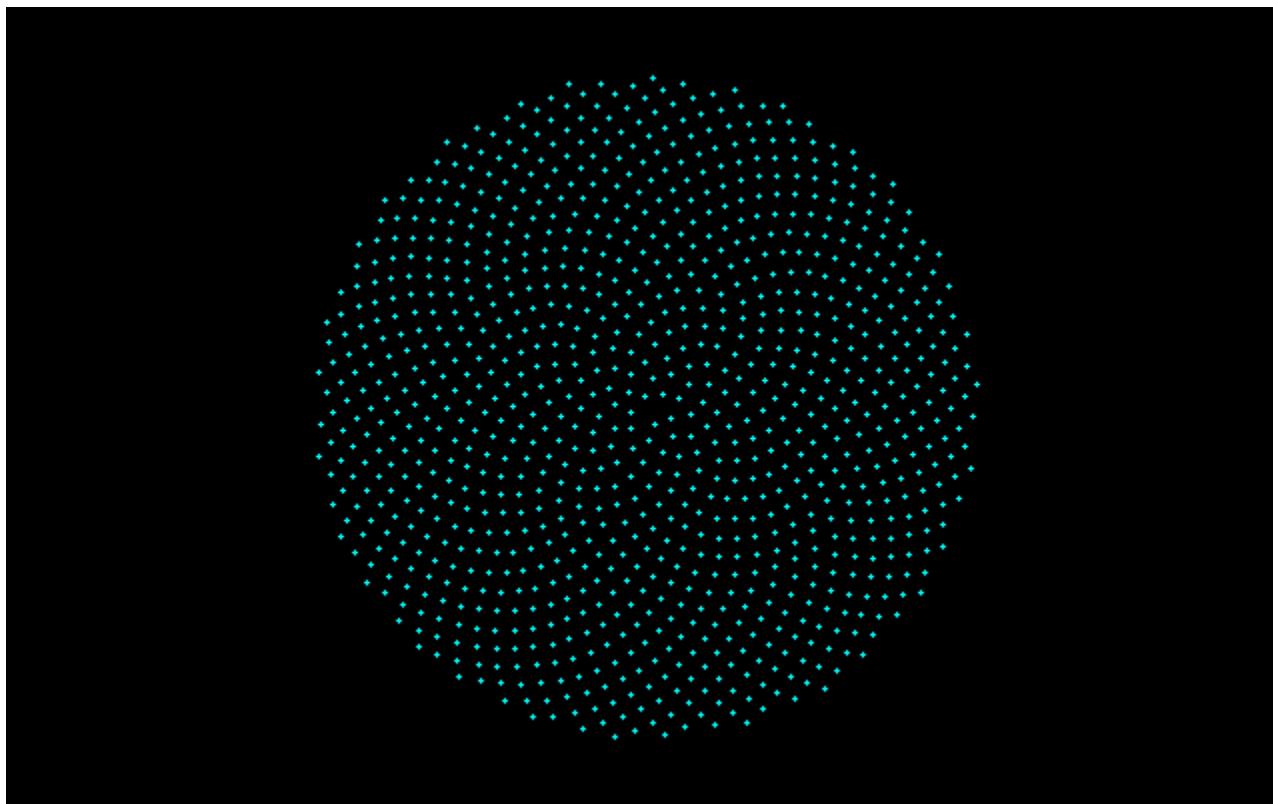
222.5

黃金角,  $\alpha=1/(\varphi+1)$   
=[0;2,1,1,1,1,1,1,1,1,· · ·]



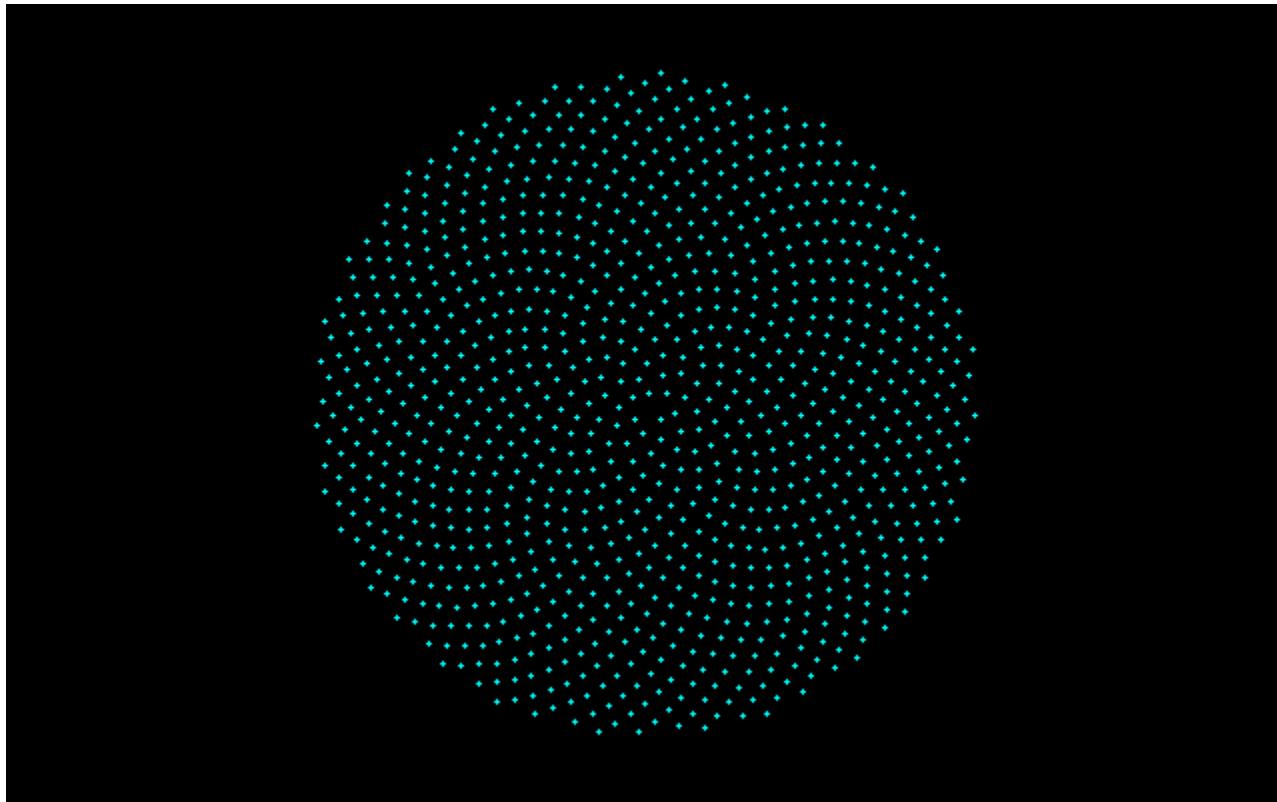
137.5

Lucas角,  $\alpha=1/(\varphi+2)$   
=[0;3,1,1,1,1,1,1,1,1, $\cdots$ ]



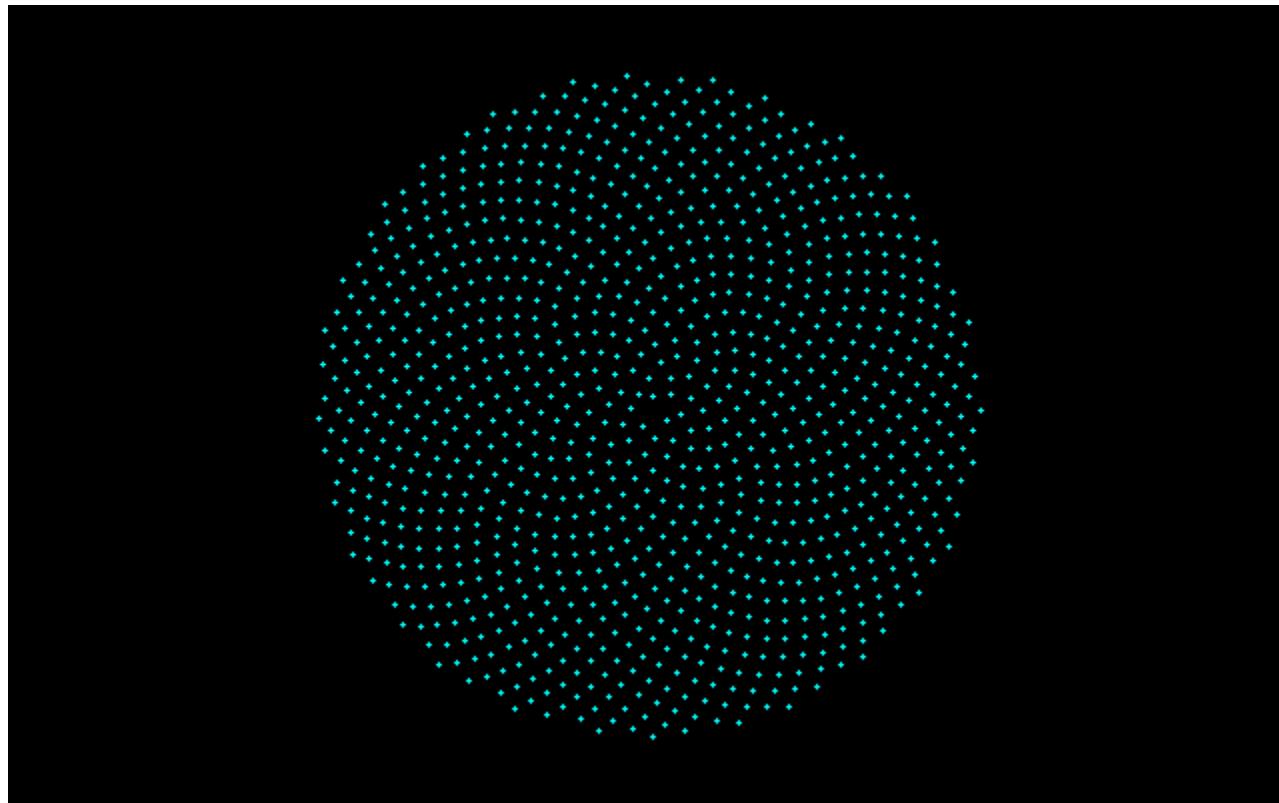
99.5

$$\alpha = 1/(\varphi + 3)$$
$$=[0;4,1,1,1,1,1,1,1,\dots]$$



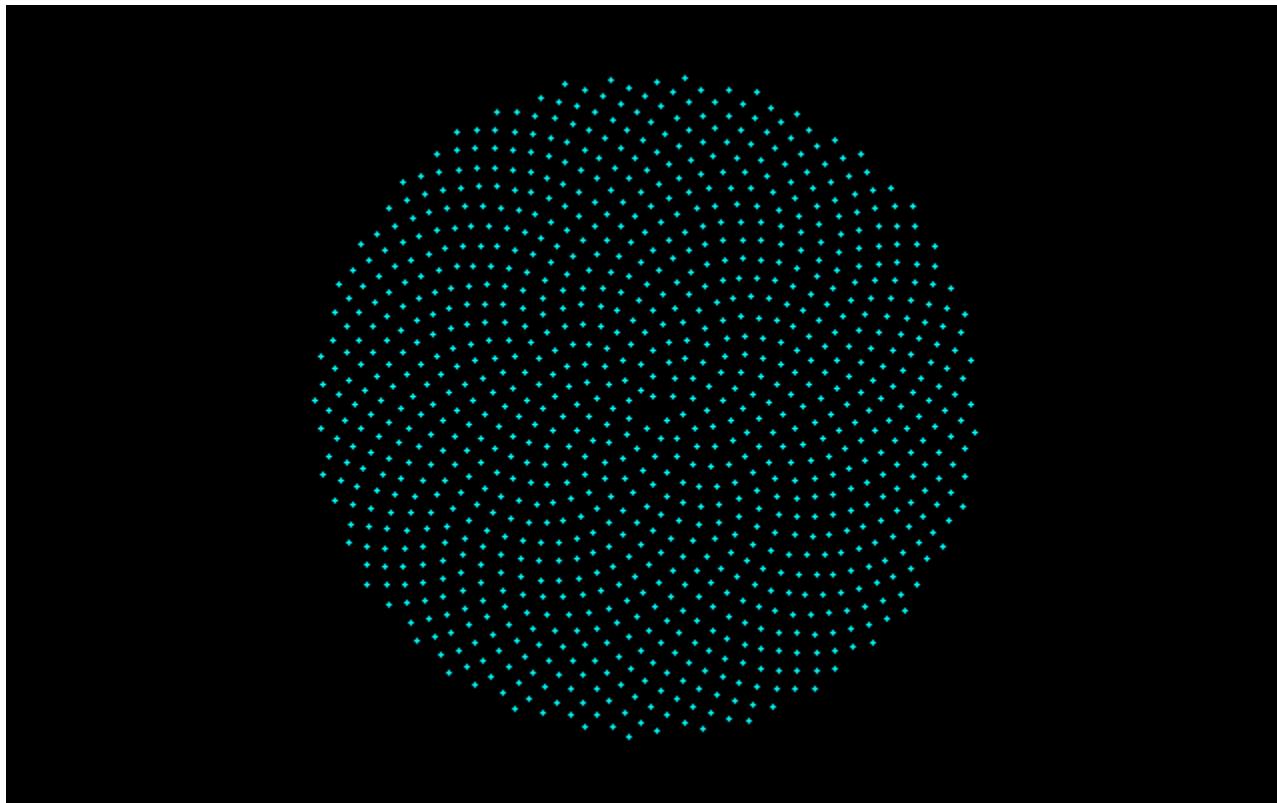
78.0

$$\alpha = 1/(\varphi + 4)$$
$$=[0;5,1,1,1,1,1,1,1,\dots]$$



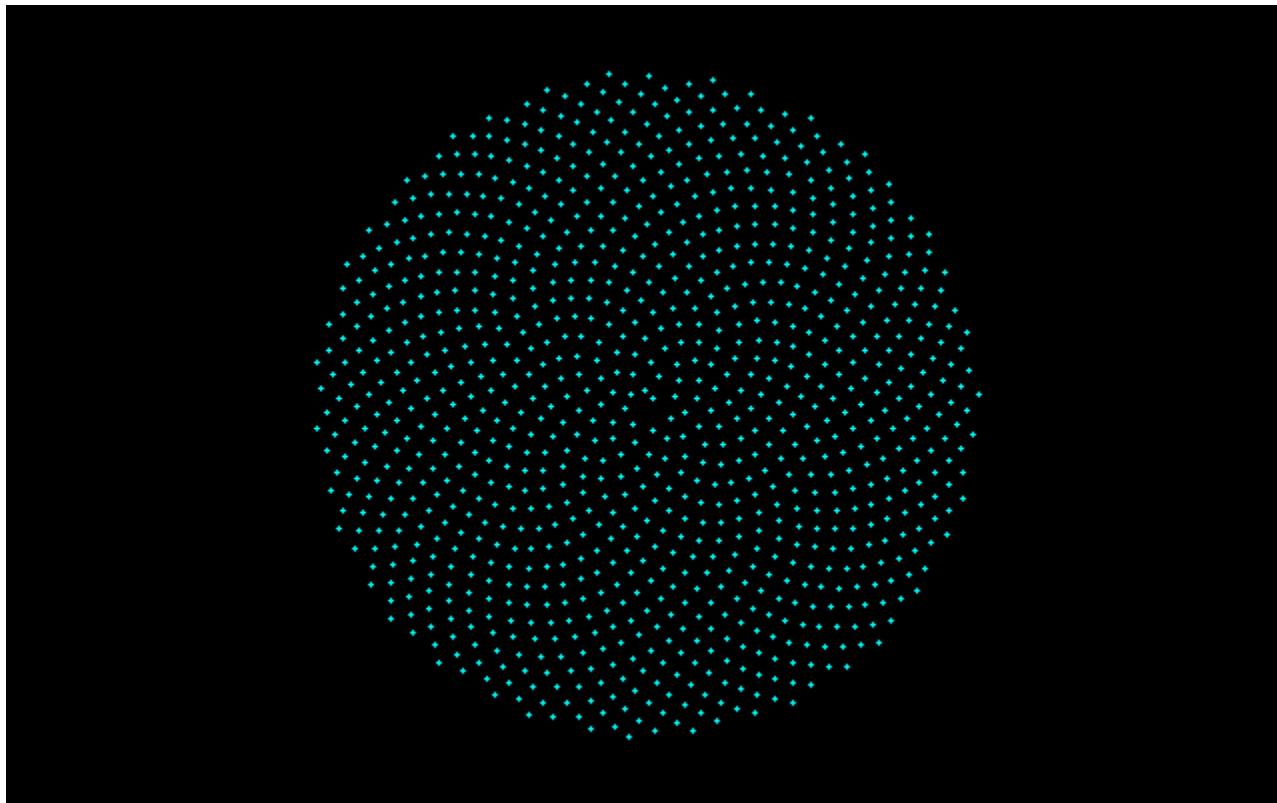
64.1

$$\alpha = 1/(\varphi + 5)$$
$$=[0;6,1,1,1,1,1,1,1,\dots]$$



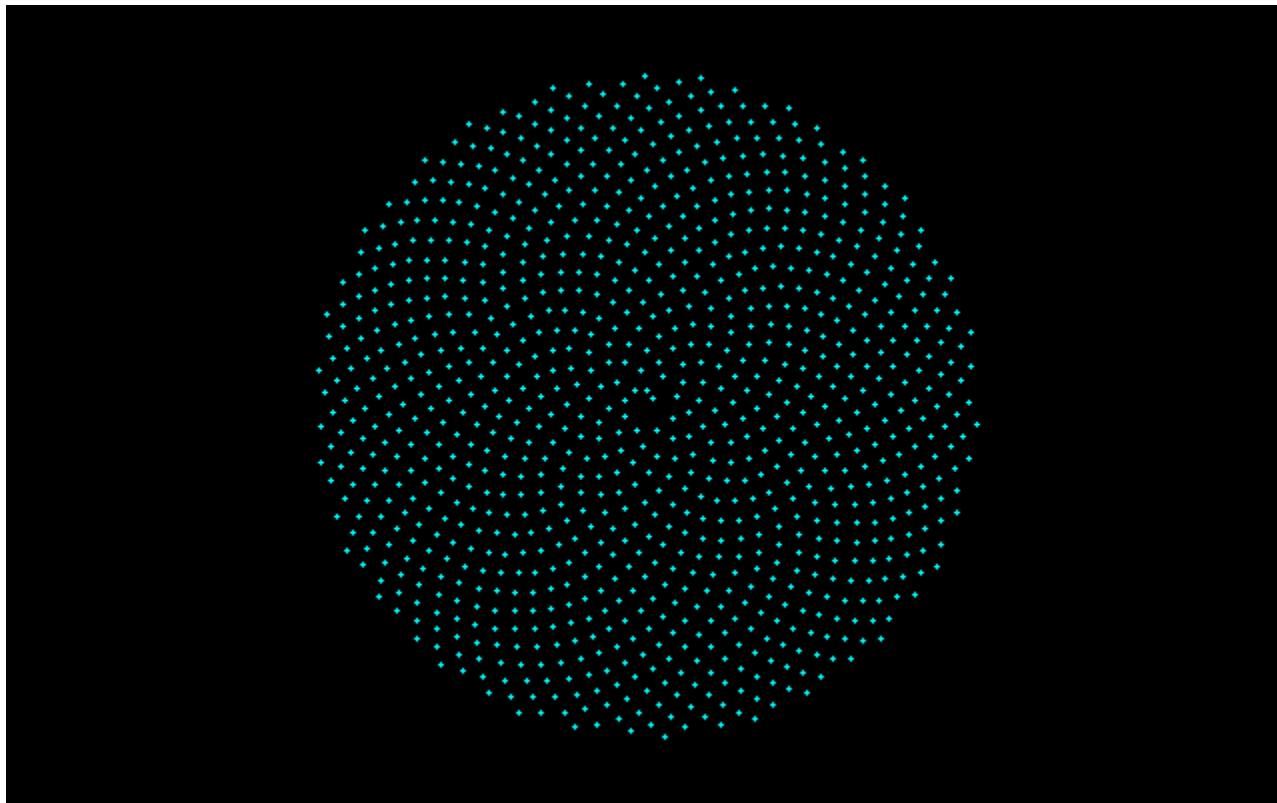
54.4

$$\alpha = 1/(\varphi + 6)$$
$$=[0;7,1,1,1,1,1,1,1,\dots]$$



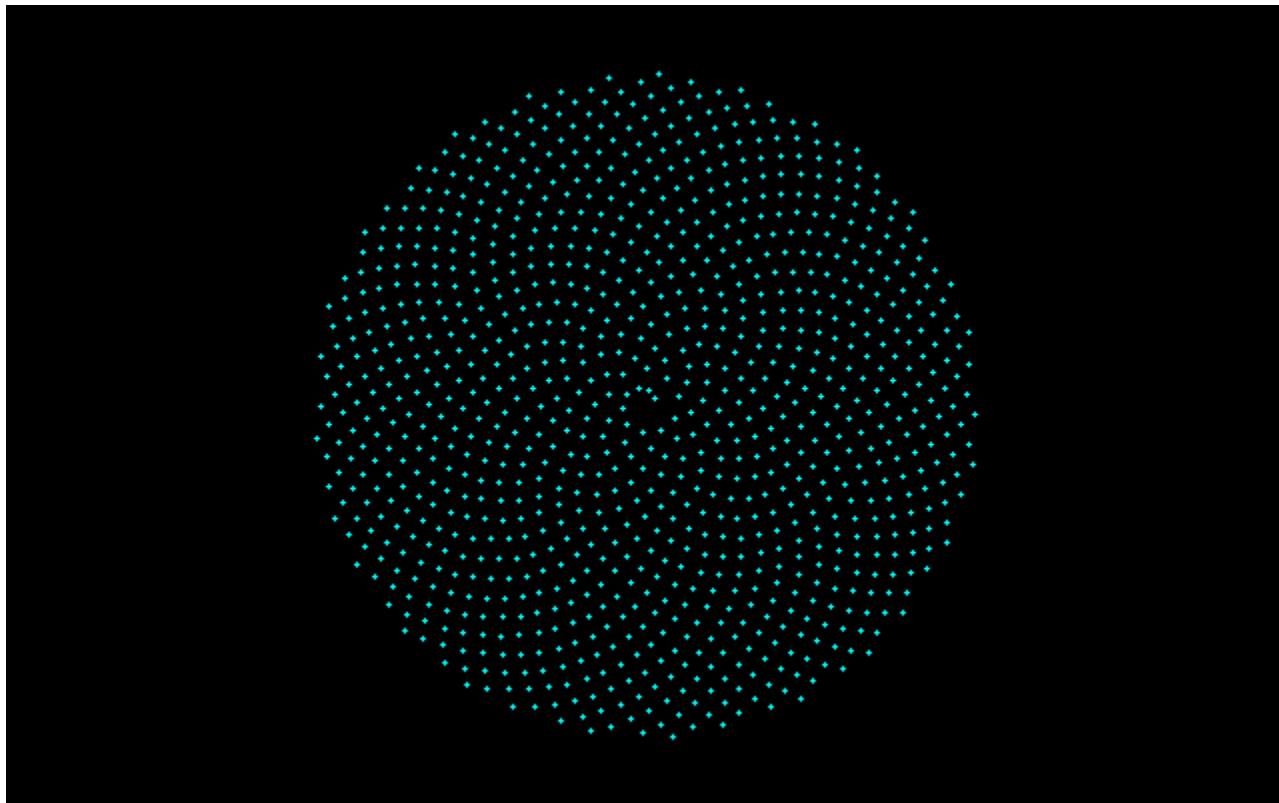
47.3

$$\alpha = 1/(\varphi + 7)$$
$$=[0;8,1,1,1,1,1,1,1,\dots]$$



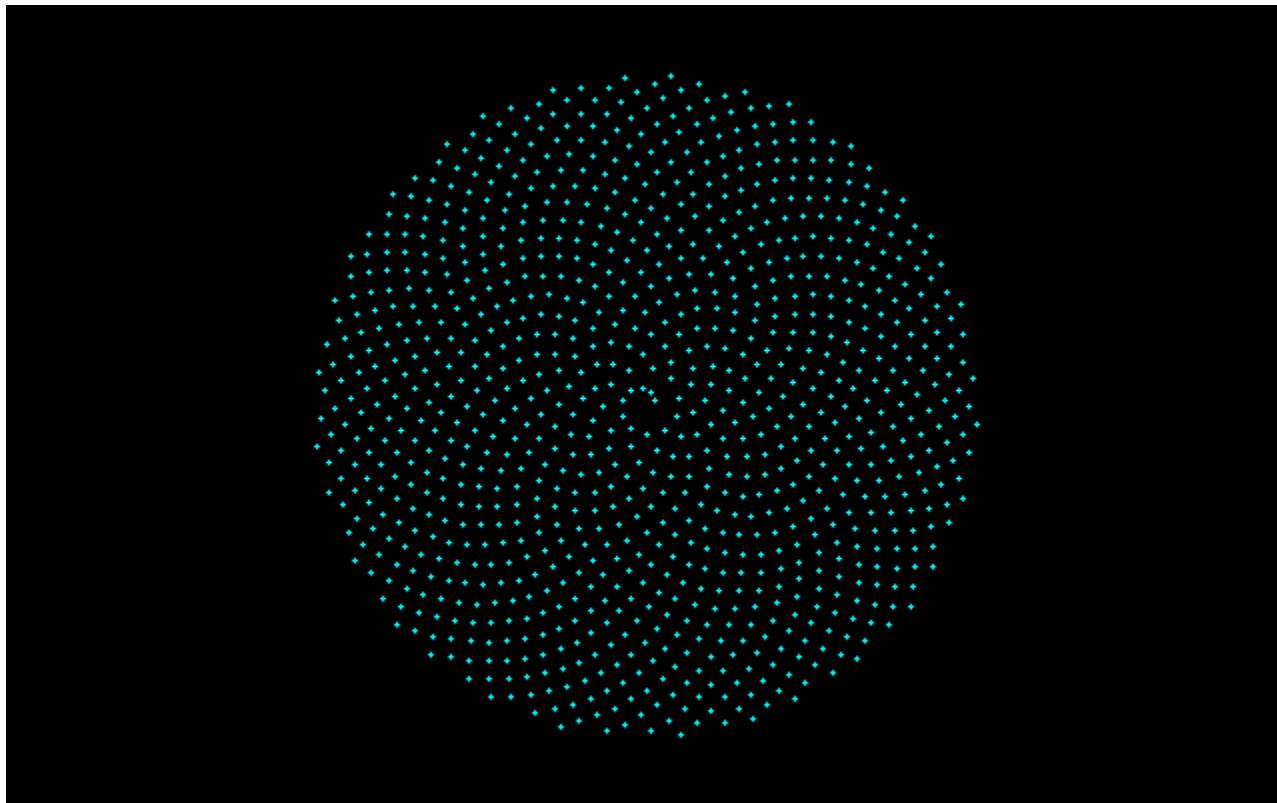
41.8

$$\alpha = 1/(\varphi + 8)$$
$$=[0;9,1,1,1,1,1,1,1,\dots]$$



37.4

$$\alpha = 1/(\varphi + 9)$$
$$=[0;10,1,1,1,1,1,1,1,\dots]$$

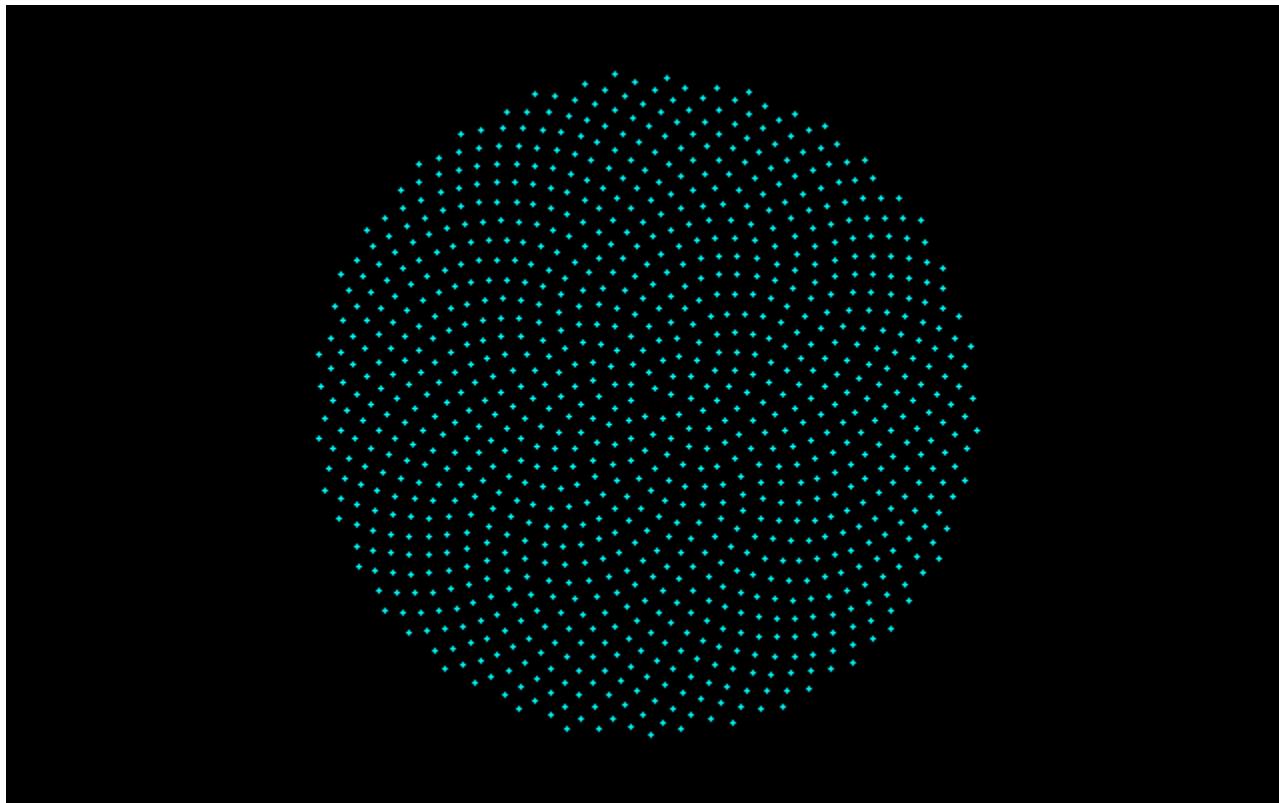


33.9

$\alpha$ の連分数展開の部分商が  
有限個を除きすべて1の場合

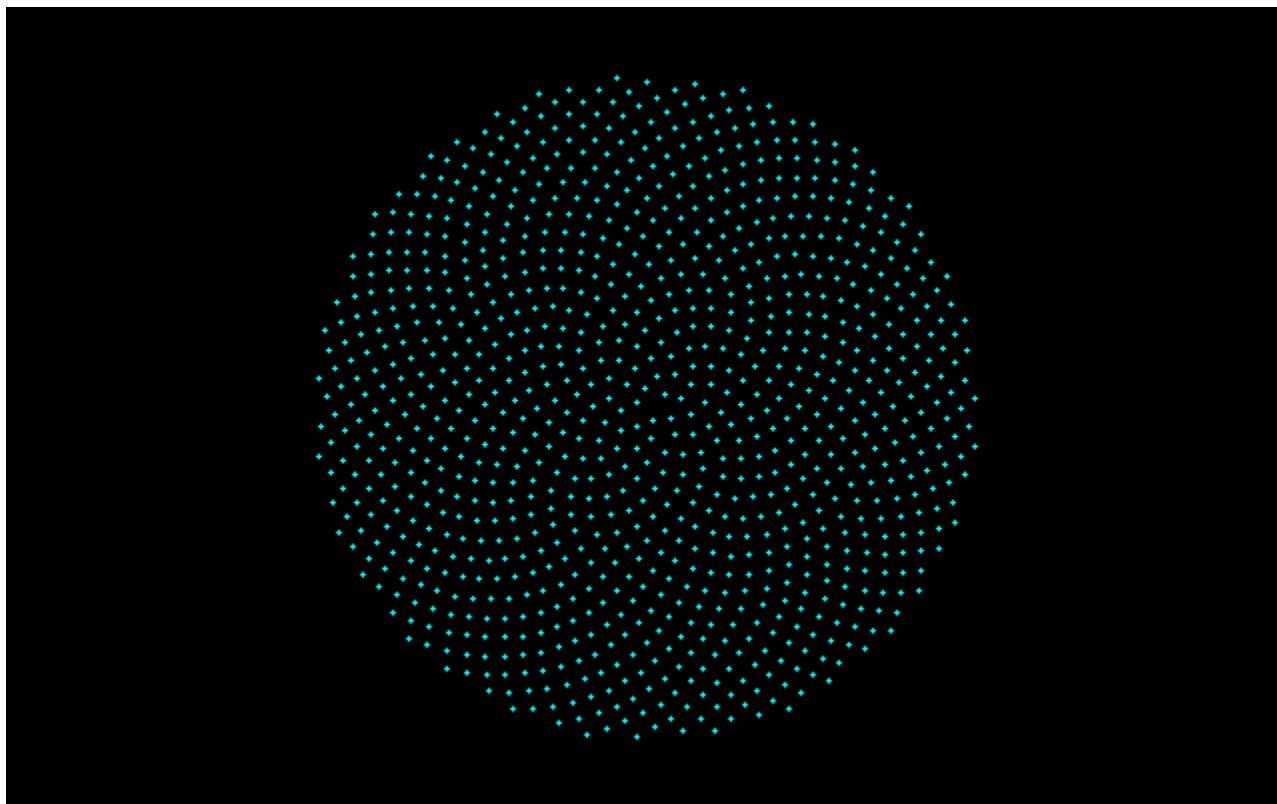
フィボナッチパターンが現れるのは  
黄金角のときには限る？

$$\alpha = 1/(3-\varphi)$$
$$=[0;1,2,1,1,1,1,1,1,\dots]$$



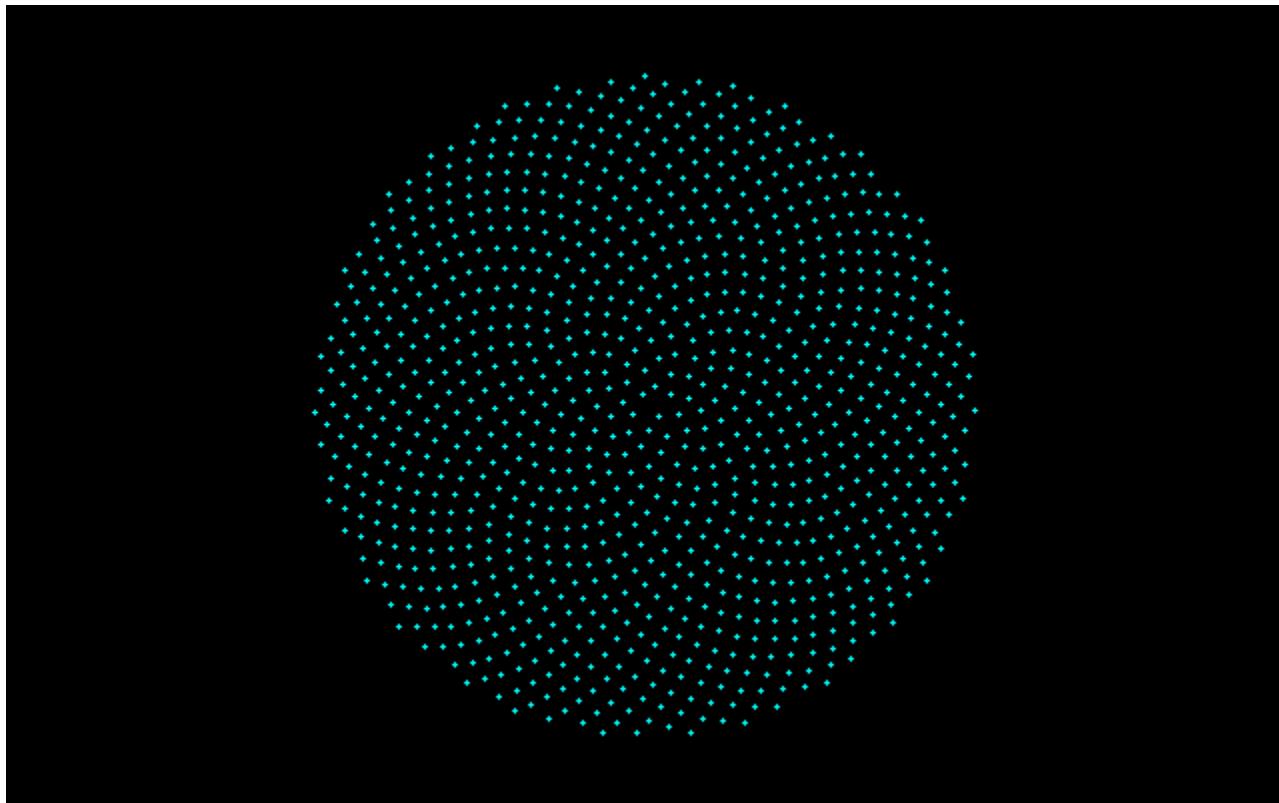
260.5

$$\alpha = 1/(4-\varphi)$$
$$=[0;2,2,1,1,1,1,1,1,\dots]$$



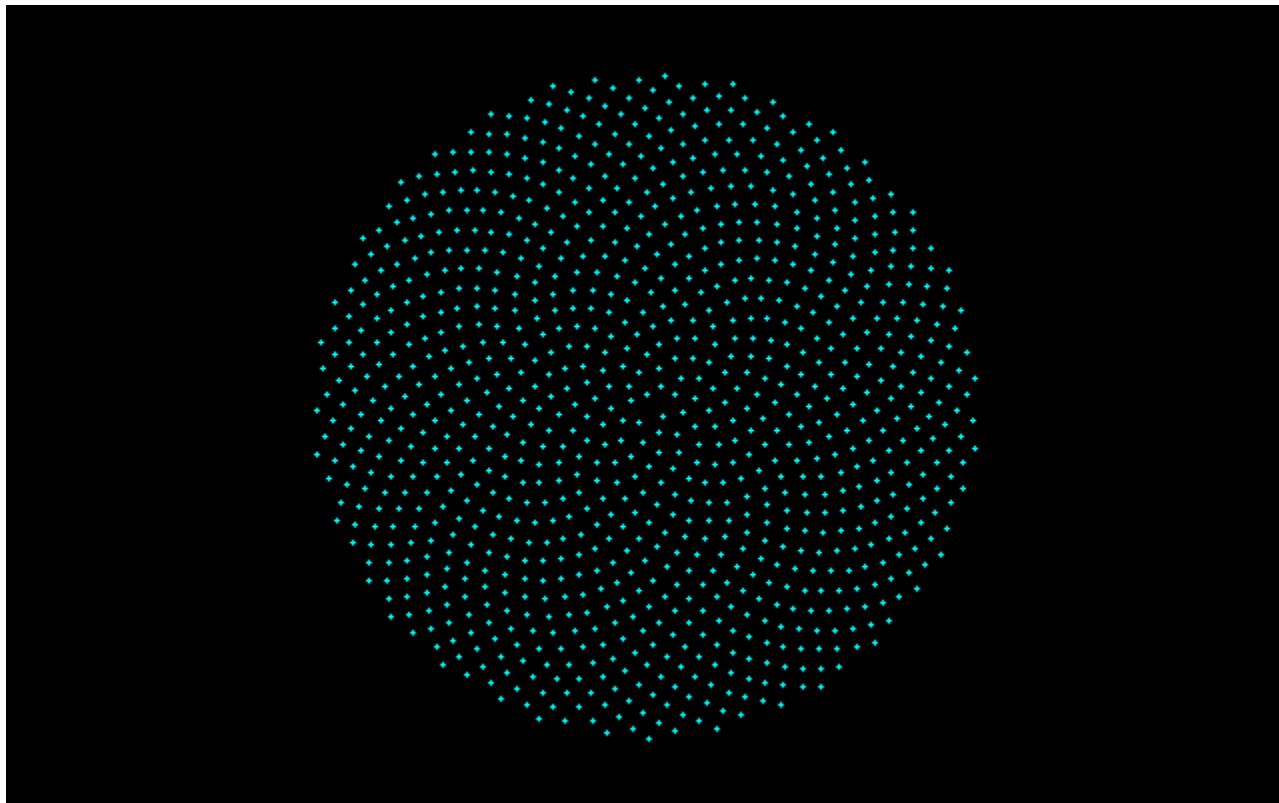
151.1

$$\alpha = 1/(5-\varphi)$$
$$=[0;3,2,1,1,1,1,1,1,\dots]$$



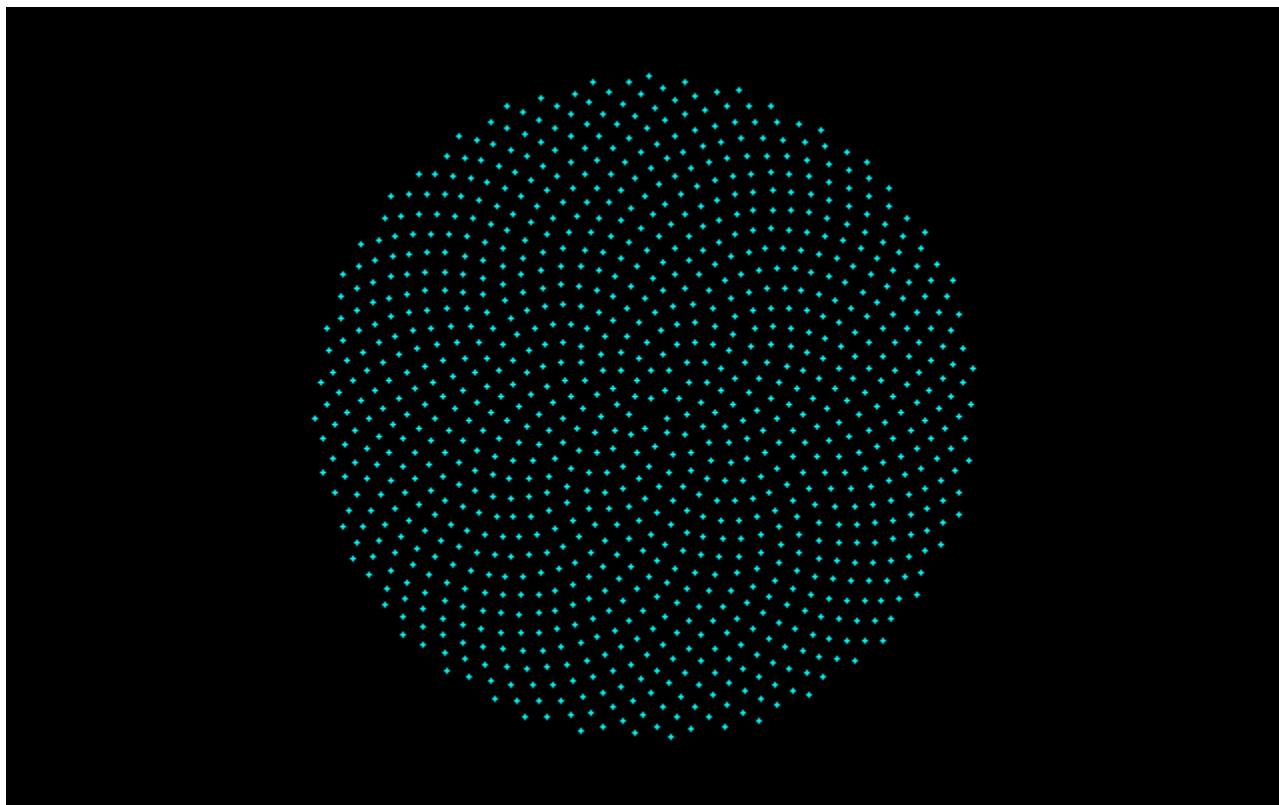
106.4

$$\alpha = 1/(6-\varphi)$$
$$=[0;4,2,1,1,1,1,1,1,\dots]$$



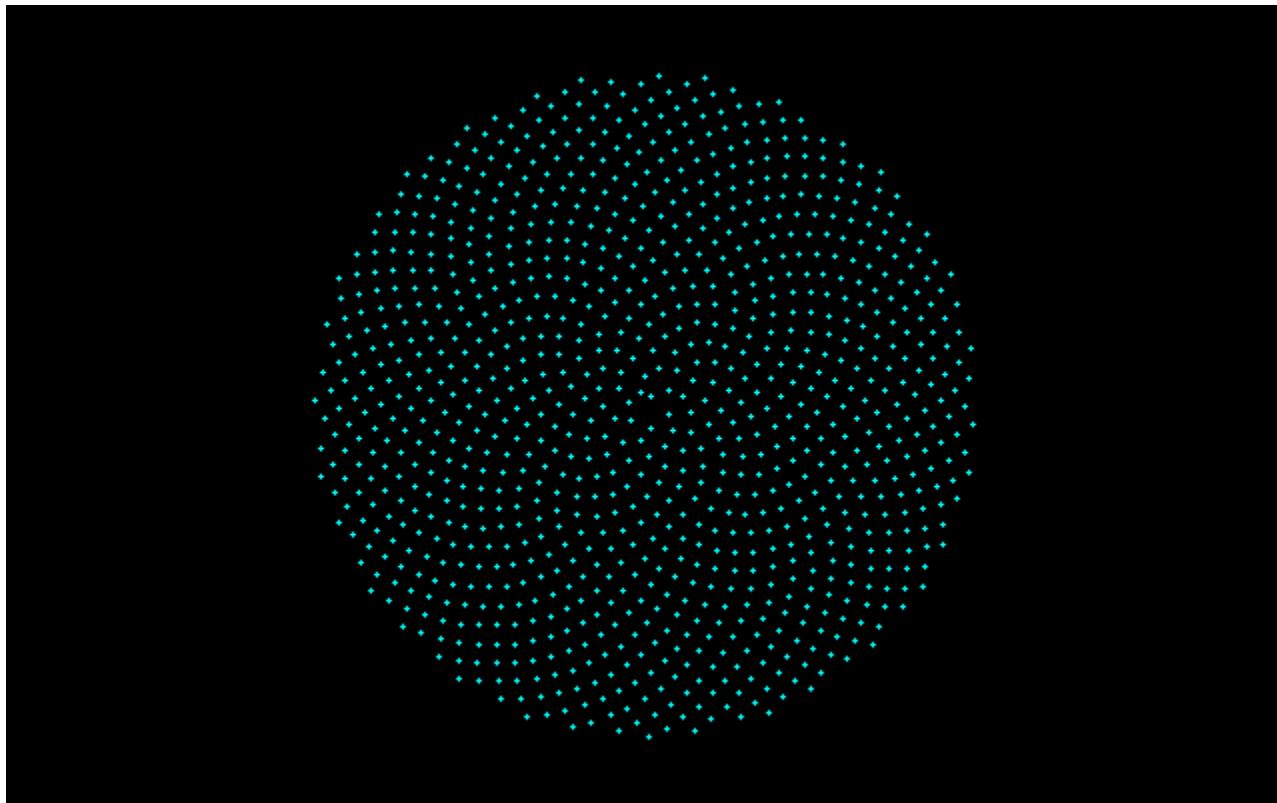
82.2

$$\alpha = 1/(7-\varphi)$$
$$=[0;5,2,1,1,1,1,1,1,\dots]$$



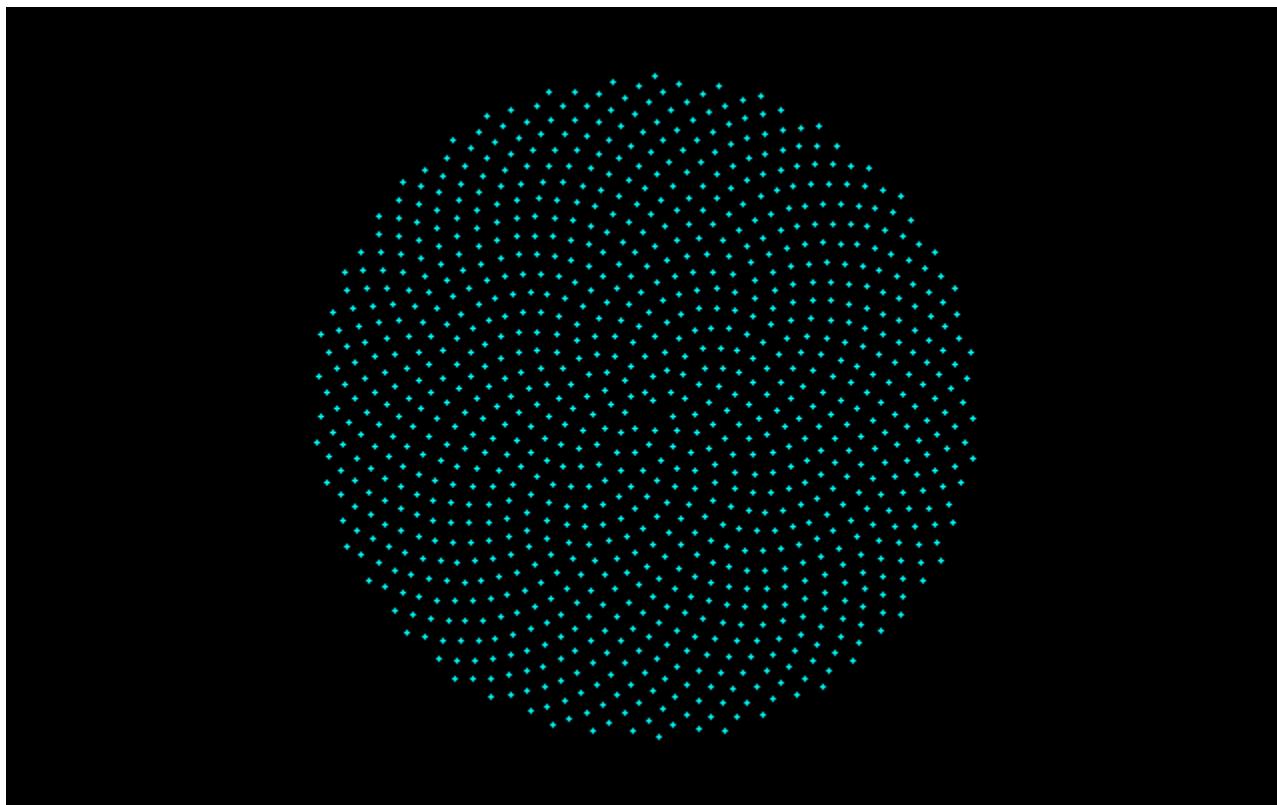
66.9

$$\alpha = 1/(8-\varphi)$$
$$=[0;3,2,1,1,1,1,1,1,\dots]$$



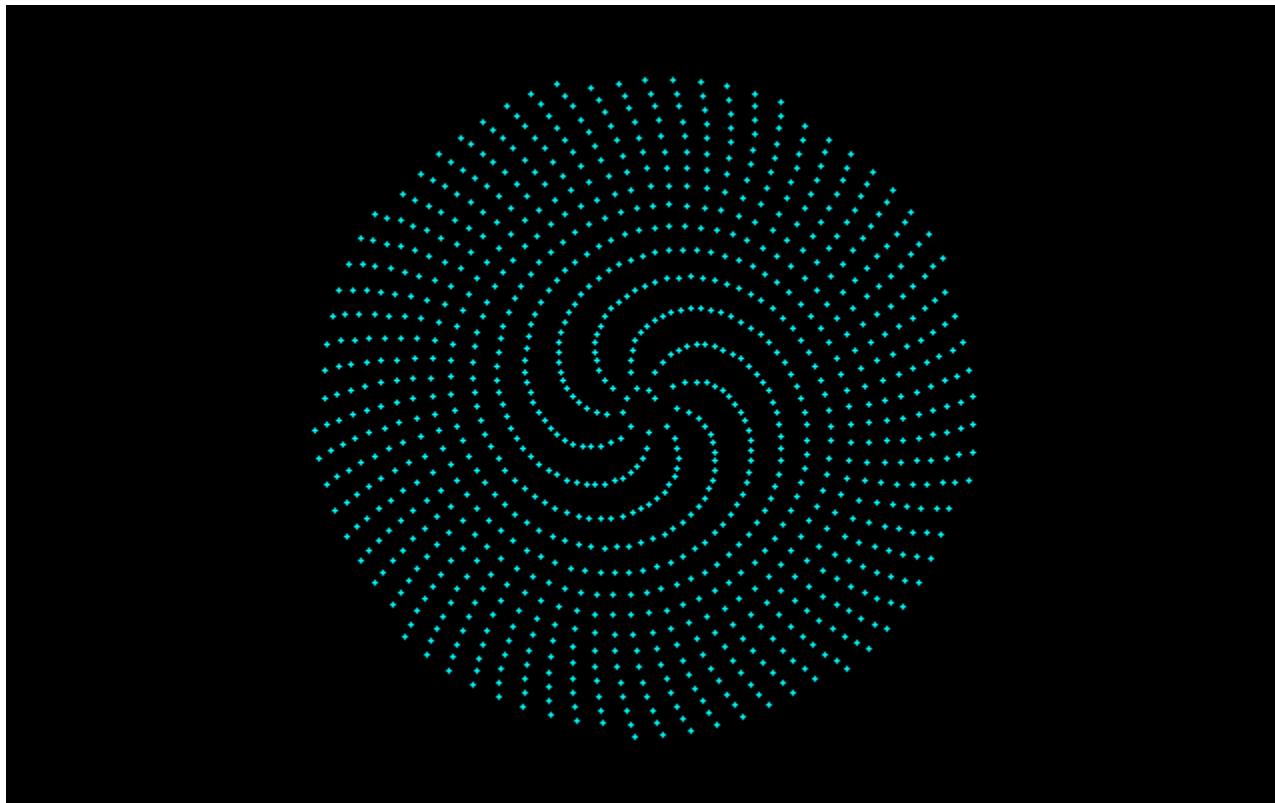
56.4

$$\alpha = 1/(9-\varphi)$$
$$=[0;7,2,1,1,1,1,1,1,\dots]$$



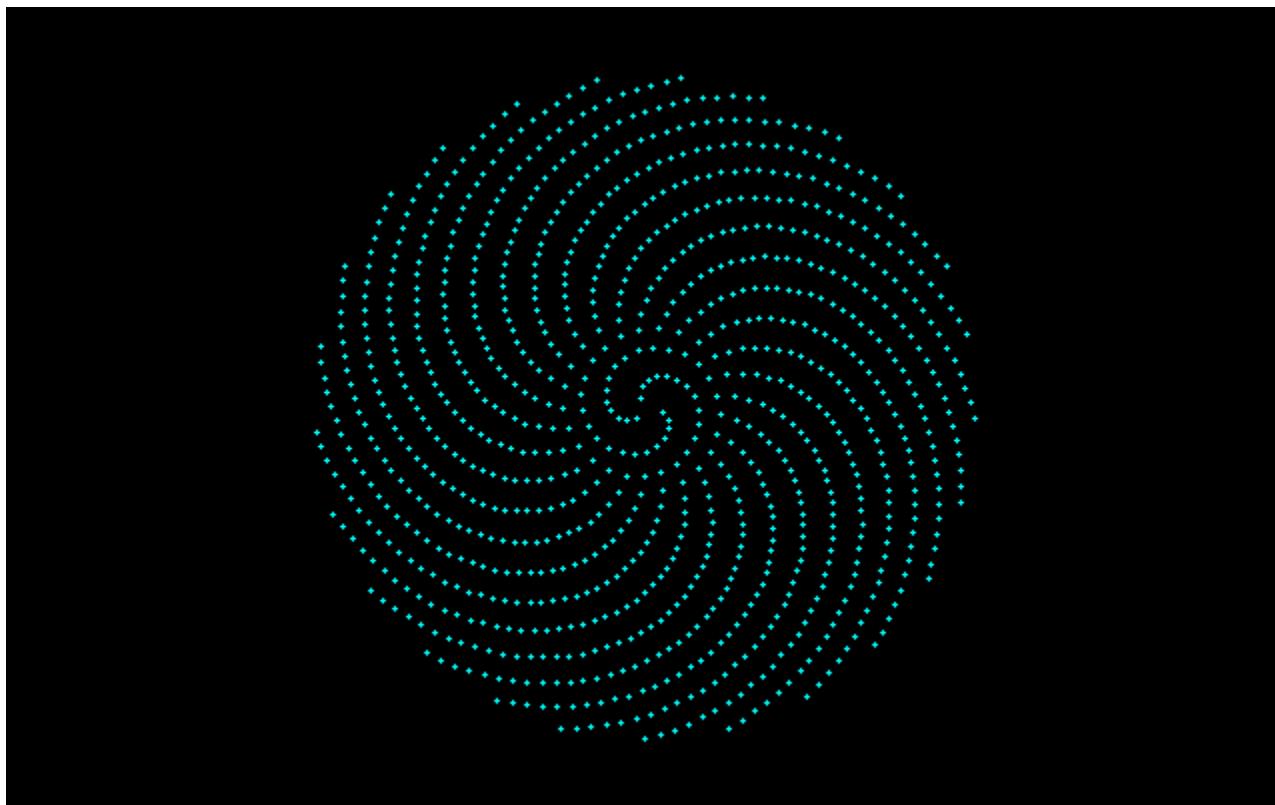
48.8

$$\begin{aligned} a &= (8\varphi + 33)/(73\varphi + 301) \\ &= [0; 9, 8, 5, 1, 1, 1, 1, 1, \dots] \end{aligned}$$



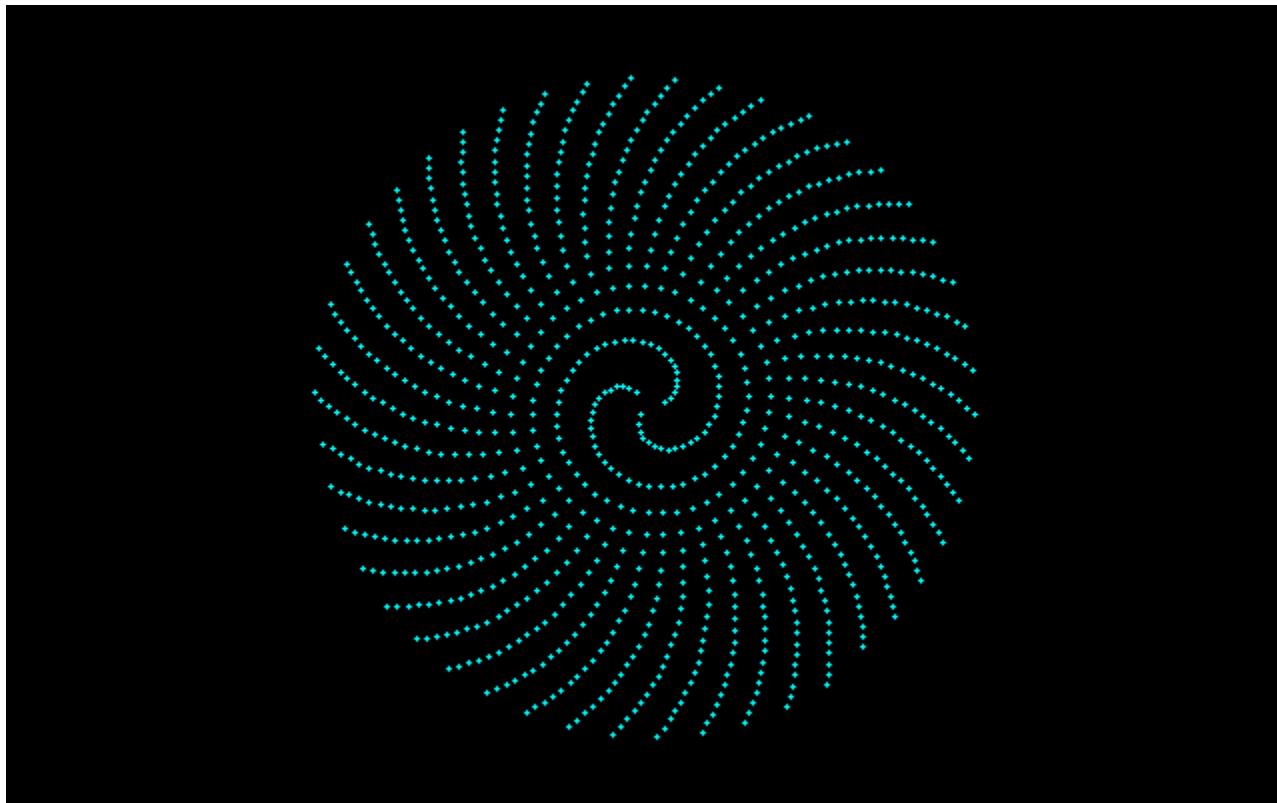
39.5

$$\alpha = \frac{8\varphi + 31}{25\varphi + 97}$$
$$= [0; 3, 7, 1, 4, 1, 1, 1, 1, \dots]$$



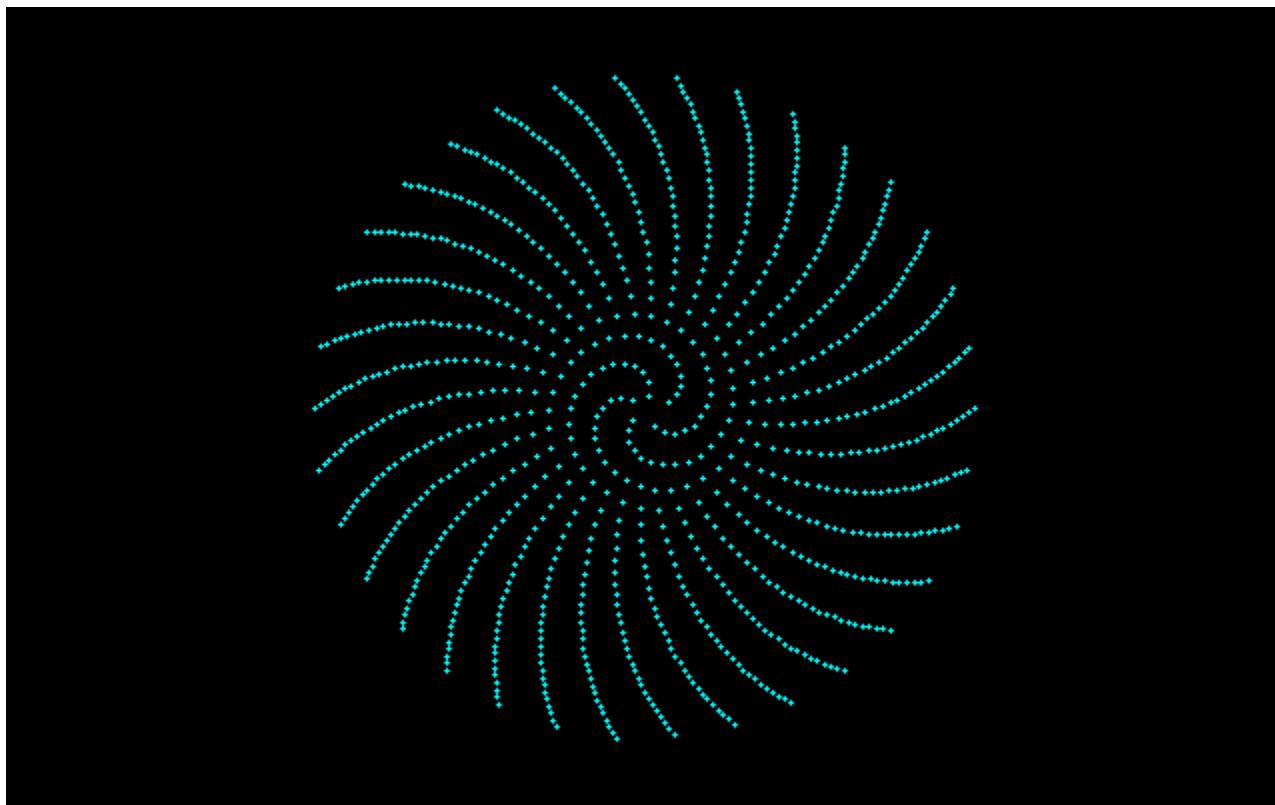
115.1

$$\alpha = (31\varphi + 157)/(46\varphi + 233)$$
$$=[0;1,2,15,6,1,1,1,1,\dots]$$



246.3

$$\begin{aligned} \alpha &= (7\varphi + 90)/(34\varphi + 437) \\ &= [0; 4, 1, 5, 1, 13, 1, 1, 1, \dots] \end{aligned}$$



# らせん上の点配置

- フィボナッチパターンが現れるのは黄金角のときに限る?・・・黄金角以外との組み合わせ
  - a) 有理数・・・フィボナッチ数・それ以外
  - b) 無理数・・・白銀比・青銅比
  - c) 超越数・・・ $\pi$ ,  $e$
- 開度 $\alpha$ の初期条件の小さい変化が引き起こす予測不可能性=カオス的な反面、構造安定となるパラメータが紙一重の差で混在している

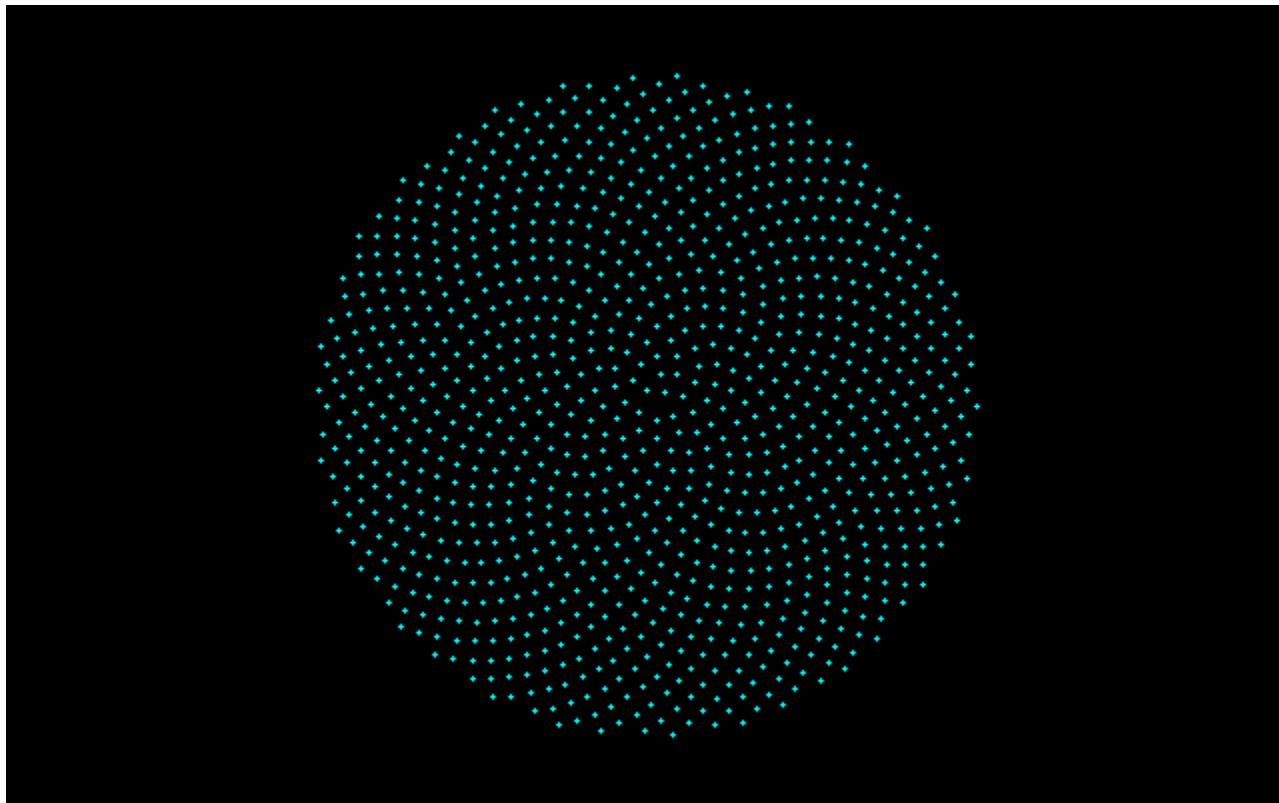
# らせん上の点配置

- フィボナッチパターンが現れる開度の分布は  
**混沌**としている
- 自然界にはフィボナッチパターンをかく乱する  
諸因子が働くと思われるが、実際にヒマワリ  
がマーガレットになることはない
- らせんも開度も一定ではありえない(背理法)

らせんの形状と開度 $\alpha$ を変化させて  
無限遠での振る舞いを調べる

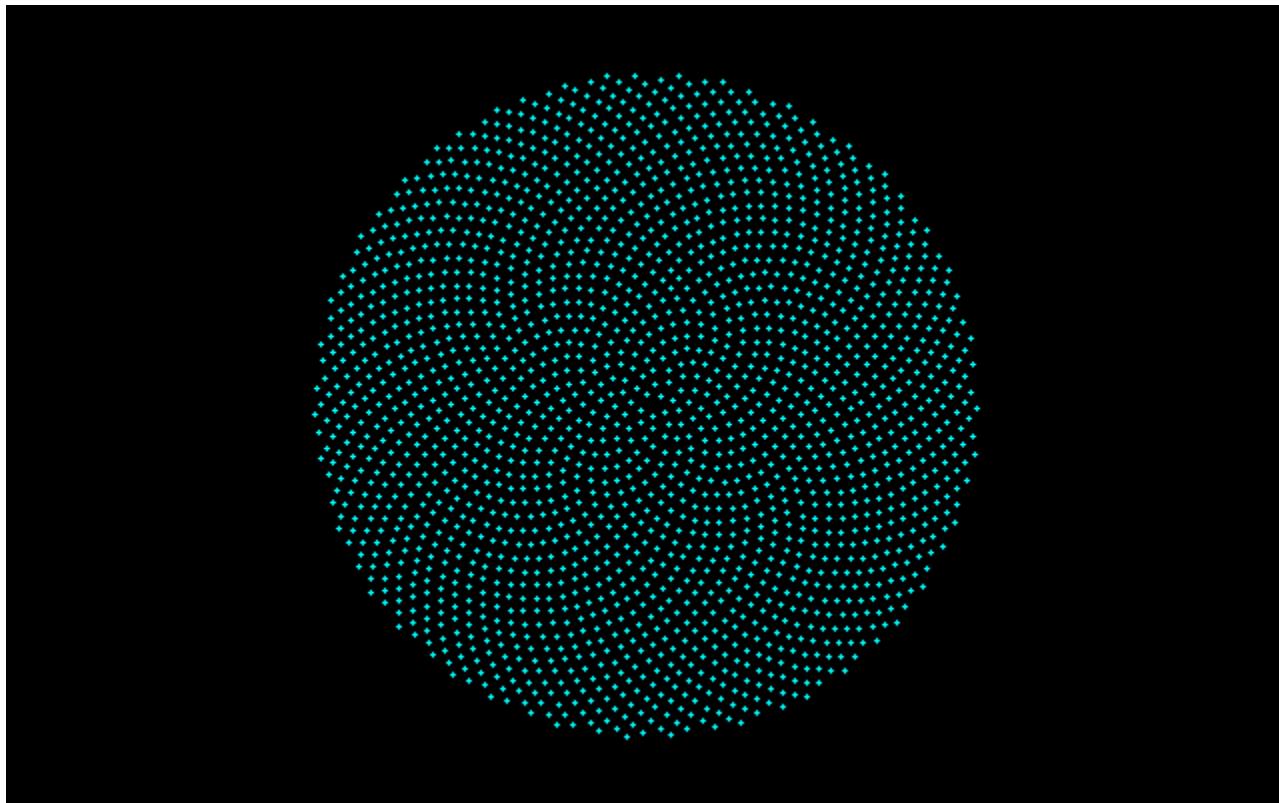
直観に反する現象を紹介  
ただし、これは植物学ではない！

$$1/\varphi = (\sqrt{5}-1)/2 = [0; 1, 1, 1, 1, 1, 1, \dots]$$



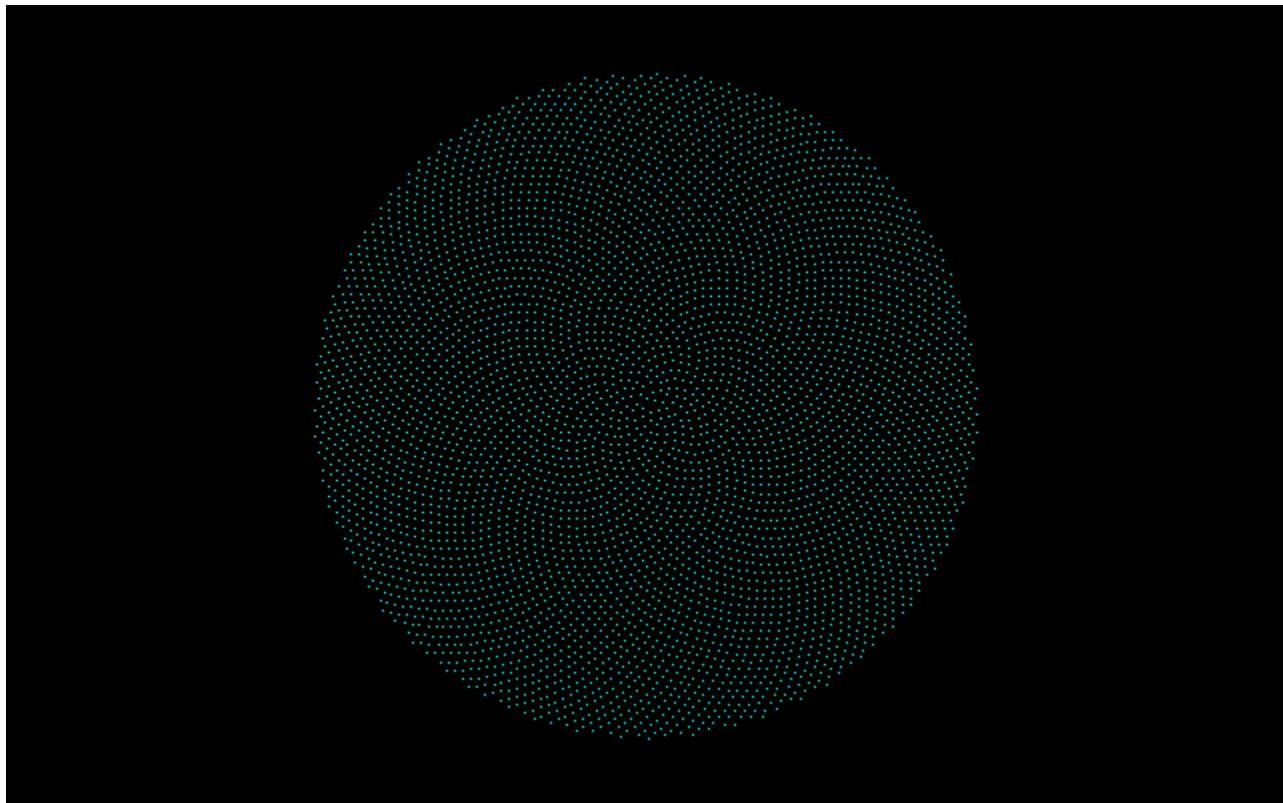
222.5

$$1/\varphi = (\sqrt{5}-1)/2 = [0; 1, 1, 1, 1, 1, 1, \dots]$$



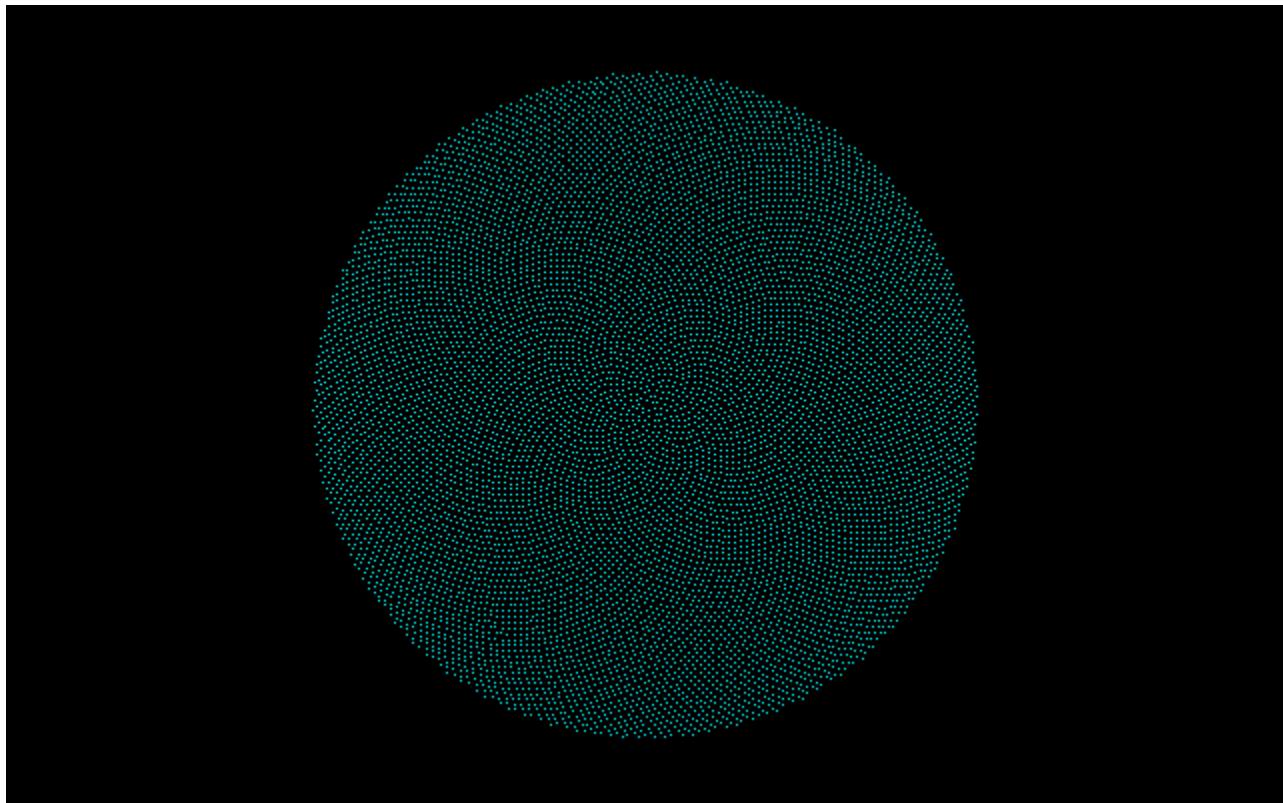
222.5

$$1/\varphi = (\sqrt{5}-1)/2 = [0; 1, 1, 1, 1, 1, 1, \dots]$$



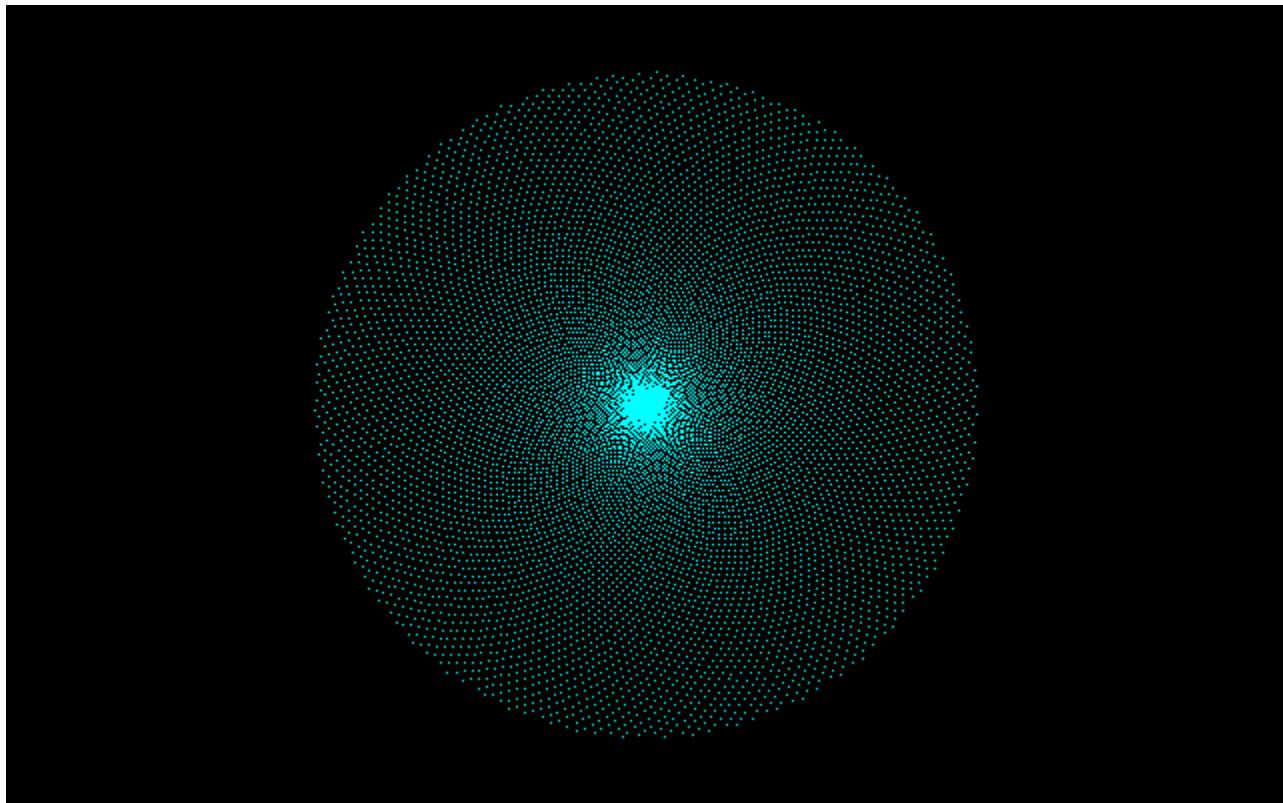
222.5

$$1/\varphi = (\sqrt{5}-1)/2 = [0; 1, 1, 1, 1, 1, 1, \dots]$$



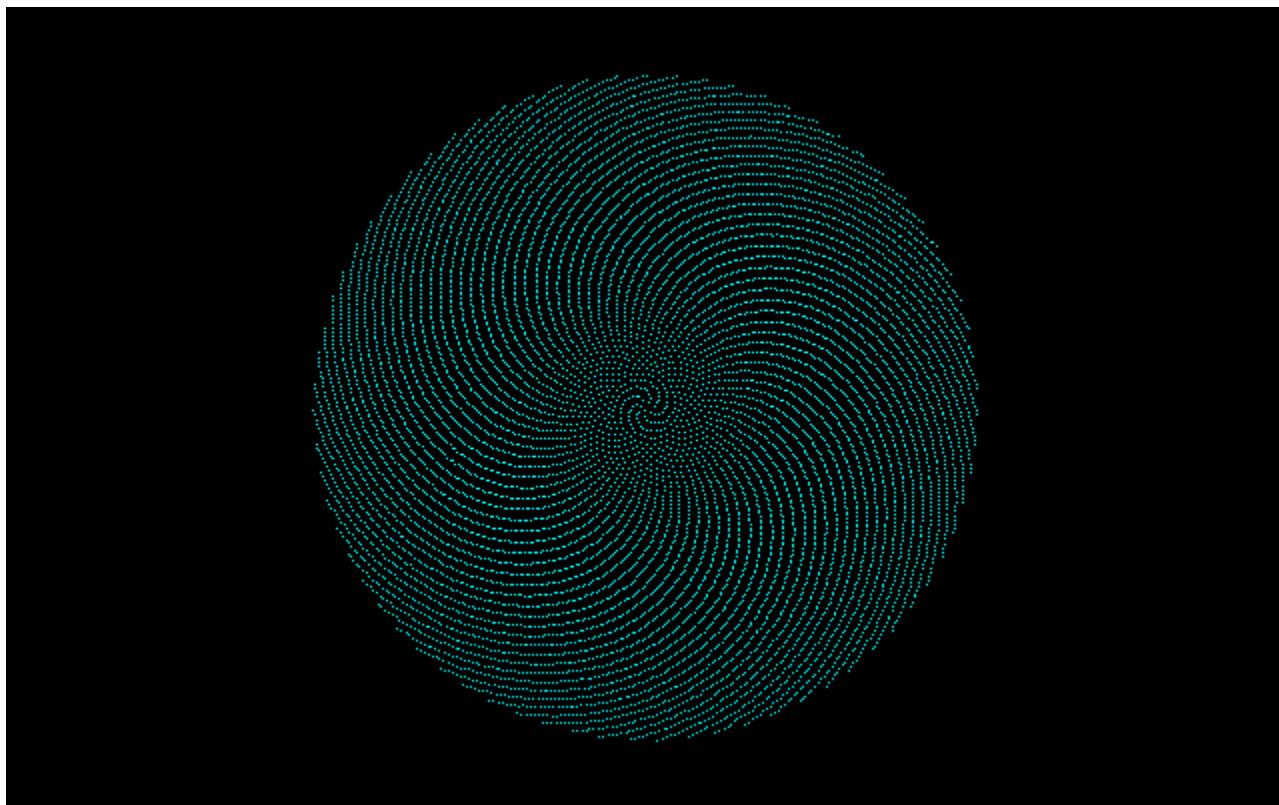
222.5

$$1/\varphi = (\sqrt{5}-1)/2 = [0; 1, 1, 1, 1, 1, 1, \dots]$$



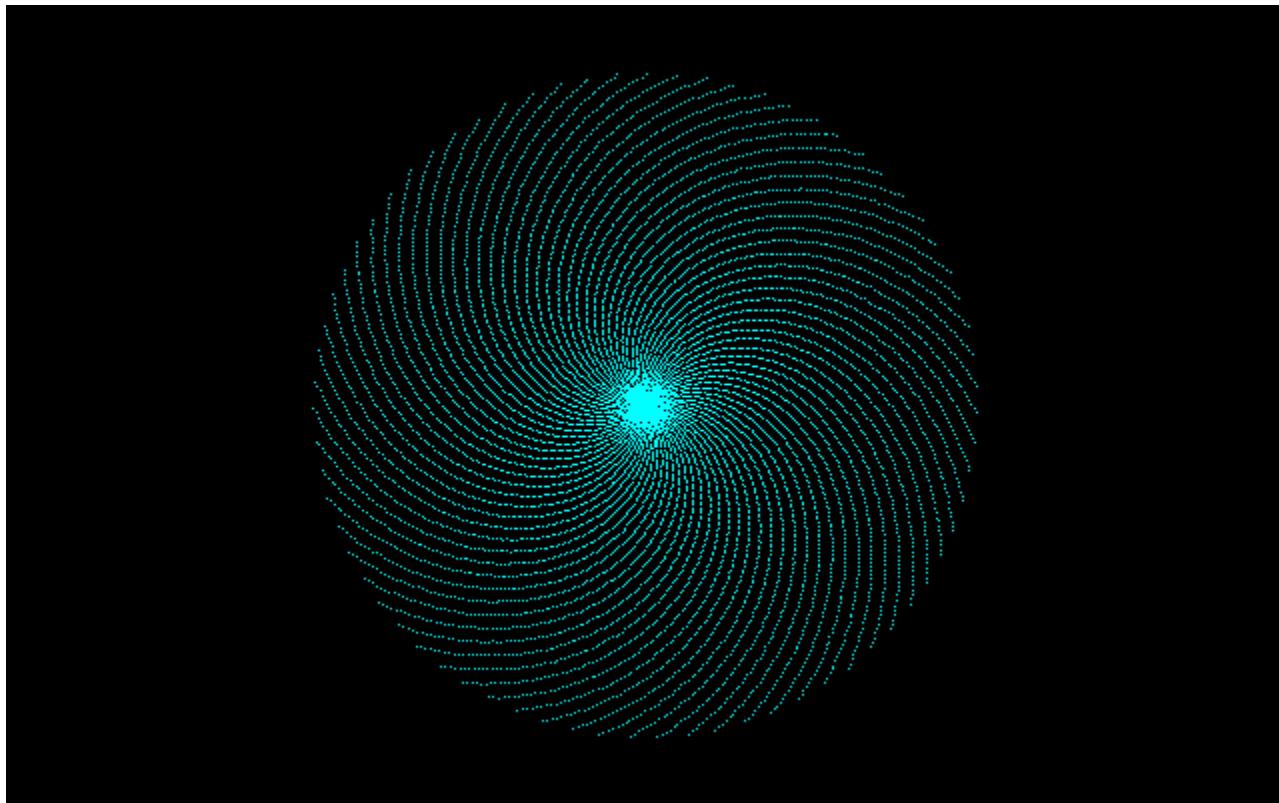
222.5

$$e-2 = [0; 1, \textcolor{red}{2}, 1, 1, \textcolor{red}{4}, 1, 1, \textcolor{red}{6}, \dots]$$



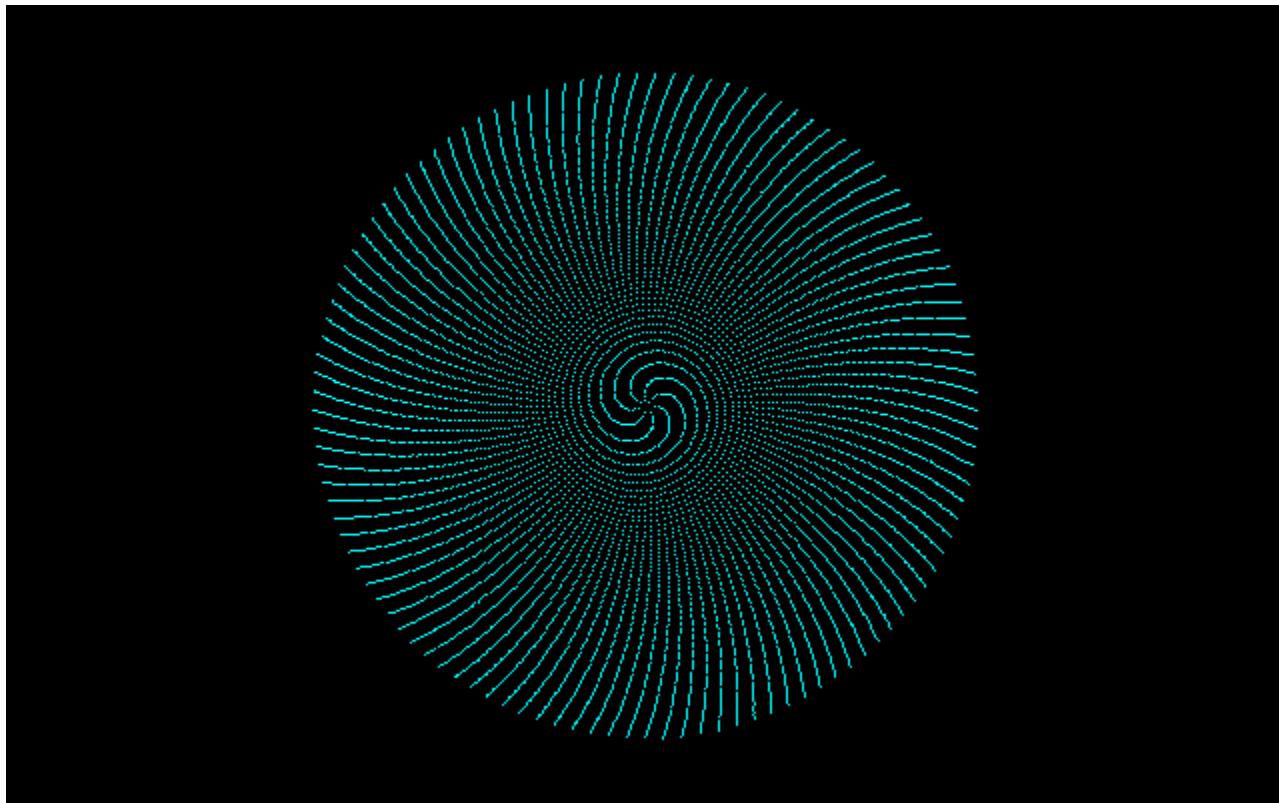
258.6

$$e-2 = [0; 1, \textcolor{red}{2}, 1, 1, \textcolor{red}{4}, 1, 1, \textcolor{red}{6}, \dots]$$



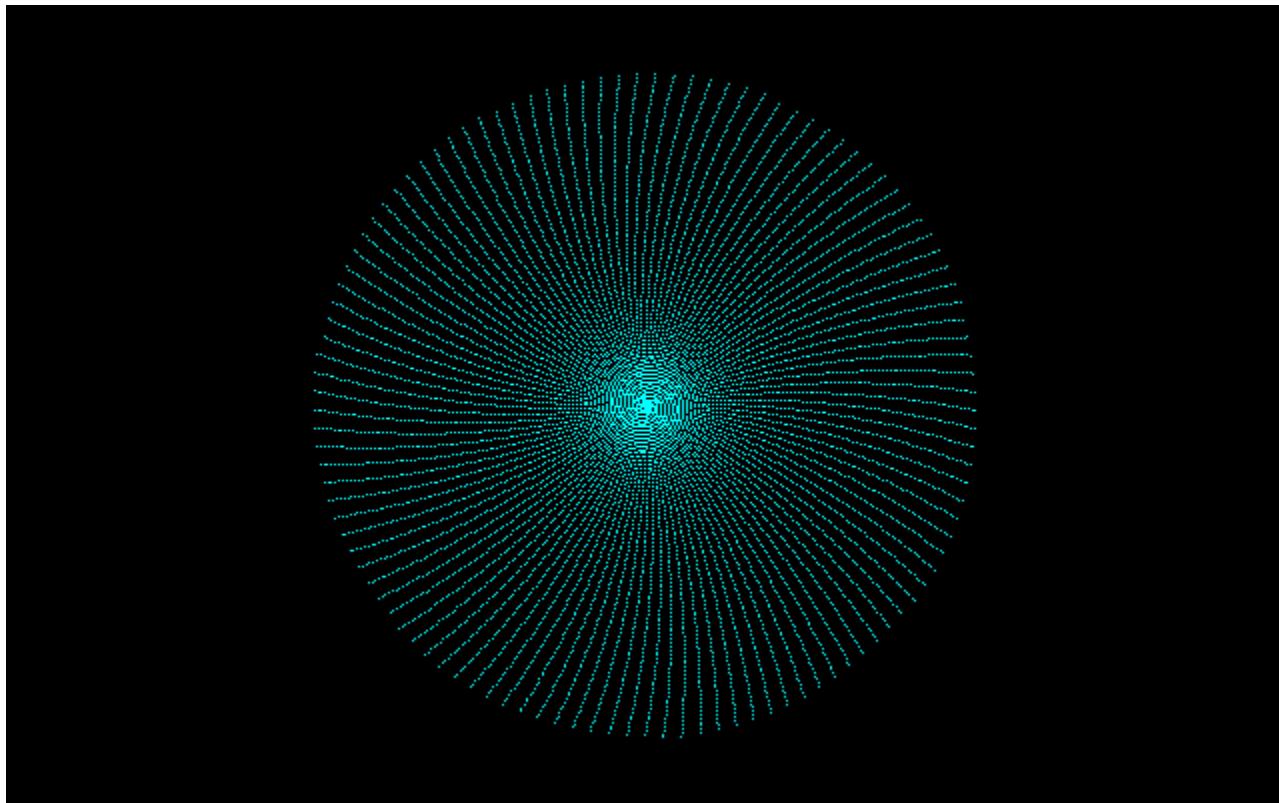
258.6

$$\pi - 3 = [0; 7, 15, 1, 292, 1, 1, 1, 2, \dots]$$



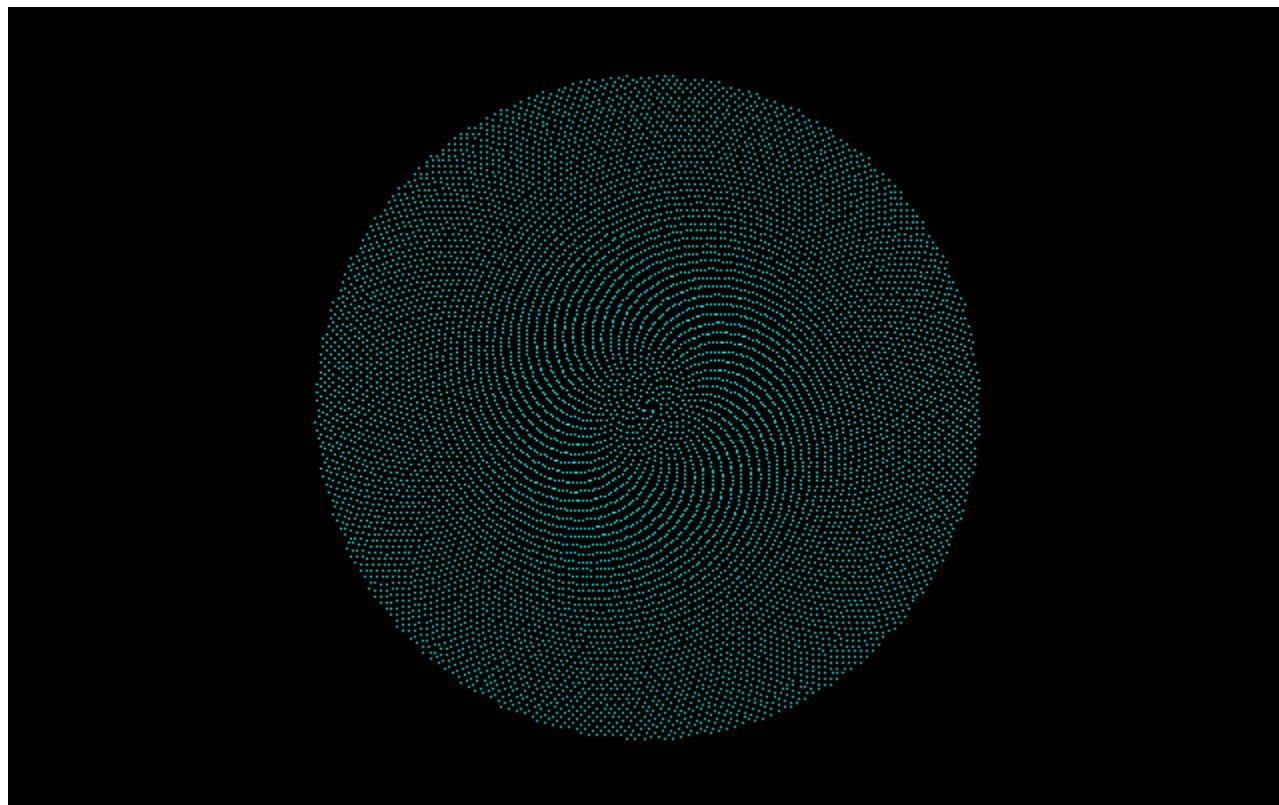
51.0

$$\pi - 3 = [0; 7, 15, 1, 292, 1, 1, 1, 2, \dots]$$



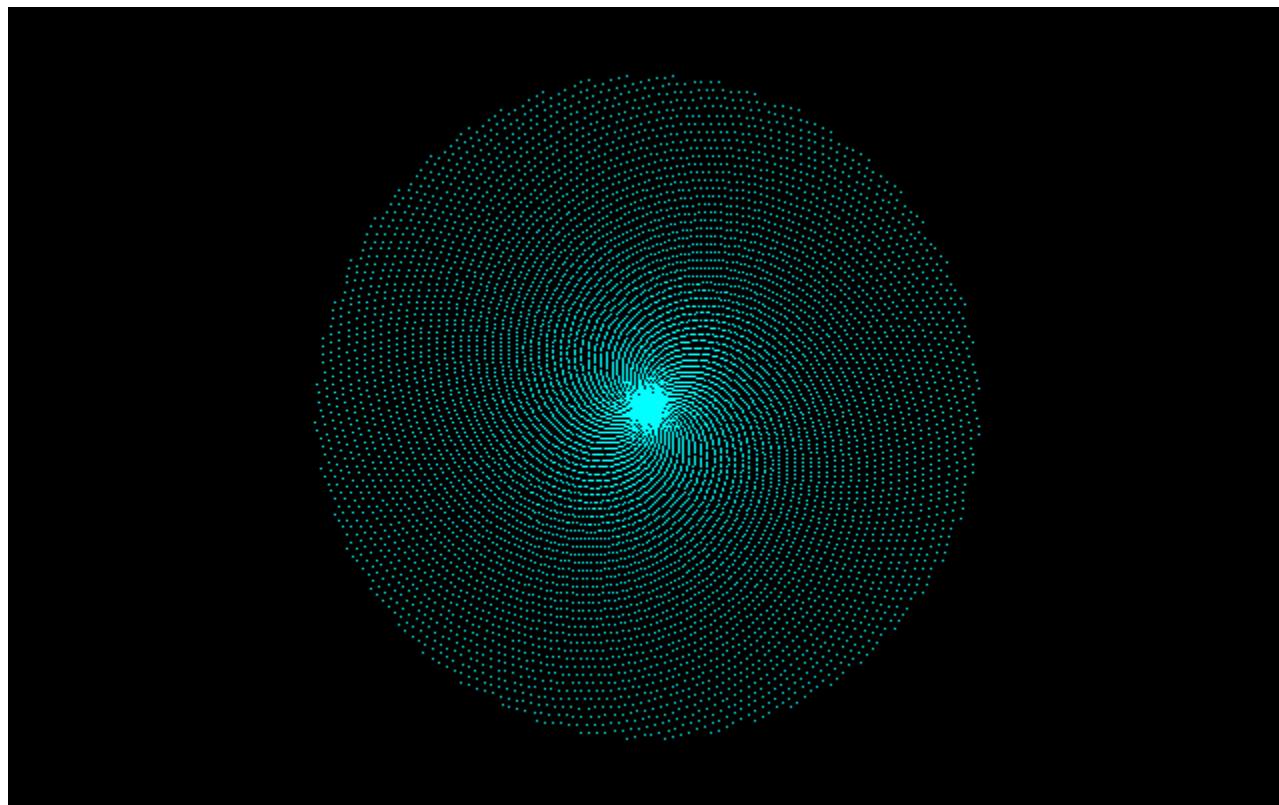
51.0

$$\alpha = \sqrt{7} - 2 = [0; 1, 1, 1, \mathbf{4}, 1, 1, 1, \mathbf{4}, \dots]$$



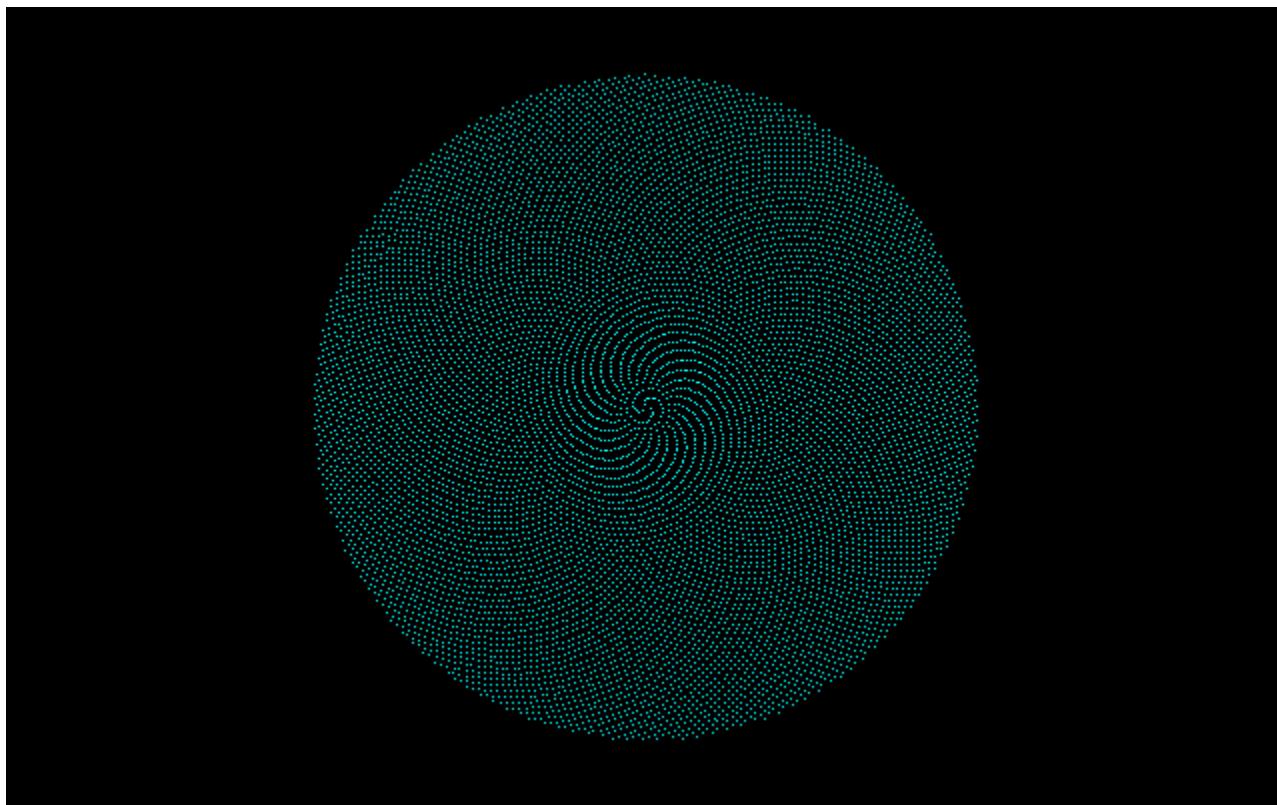
232.5

$$\alpha = \sqrt{7} - 2 = [0; 1, 1, 1, \mathbf{4}, 1, 1, 1, \mathbf{4}, \dots]$$



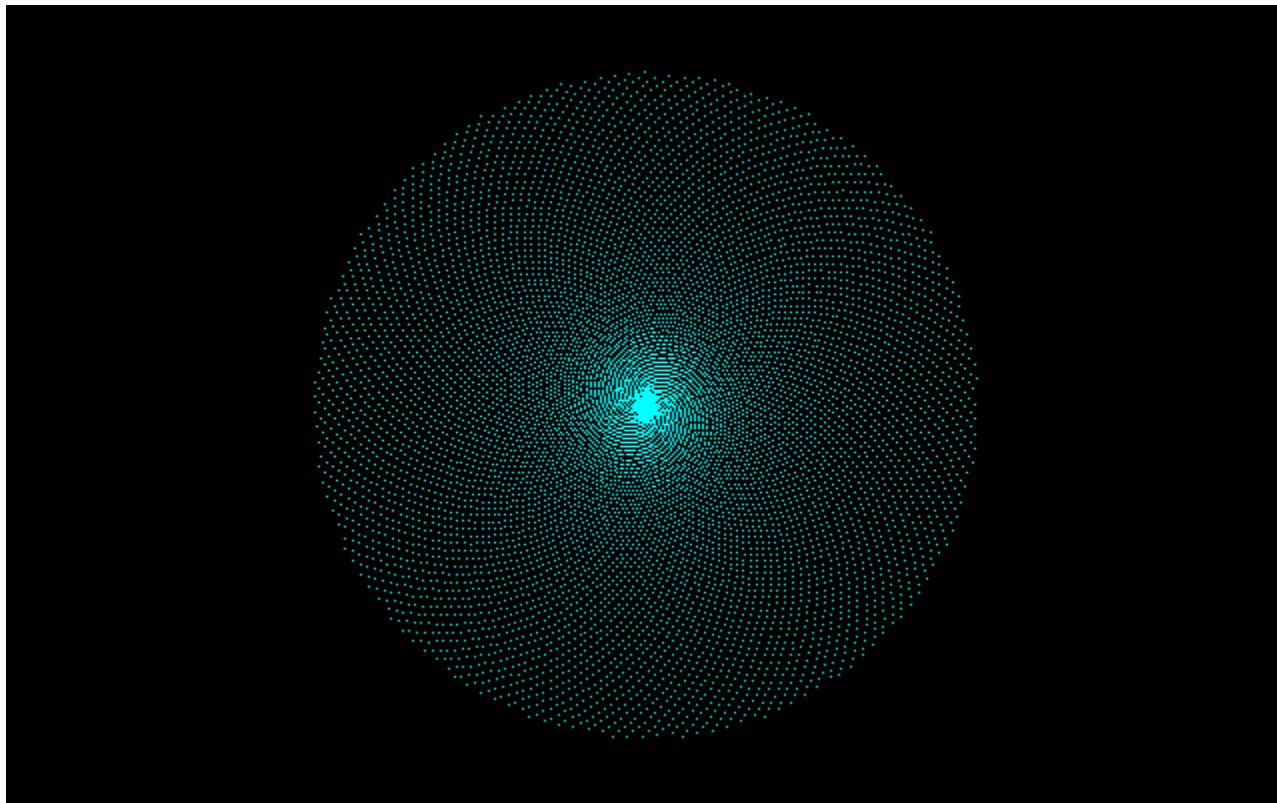
232.5

$$\alpha = \frac{8\varphi + 31}{25\varphi + 97}$$
$$=[0;3,7,1,4,1,1,1,1,\dots]$$



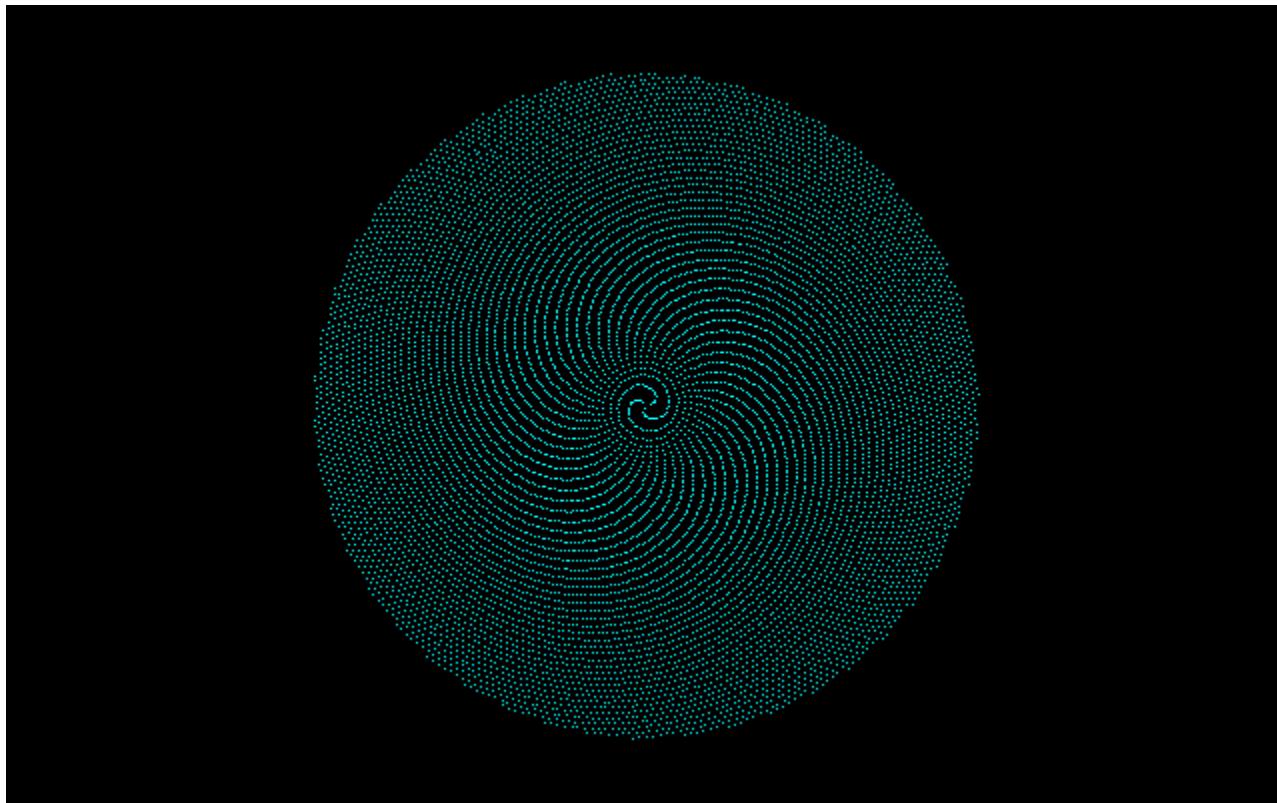
115.1

$$\alpha = \frac{8\varphi + 31}{25\varphi + 97}$$
$$=[0;3,7,1,4,1,1,1,1,\dots]$$



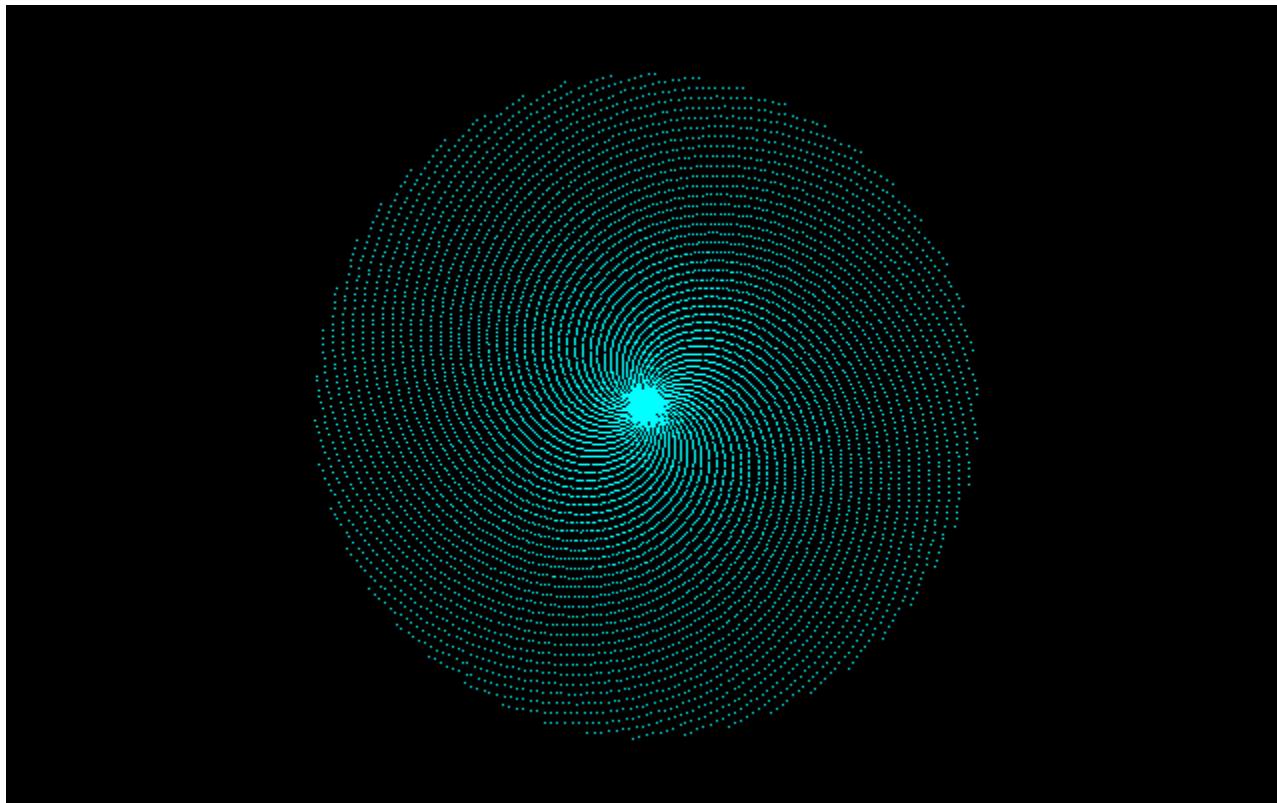
115.1

$$\alpha = \frac{31\varphi + 157}{46\varphi + 233}$$
$$=[0; \textcolor{red}{1, 2, 15, 6}, 1, 1, 1, 1, \dots]$$



246.3

$$\alpha = \frac{31\varphi + 157}{46\varphi + 233}$$
$$=[0; \textcolor{red}{1, 2, 15, 6}, 1, 1, 1, 1, \dots]$$



246.3

# らせん上の点配置

- フェルマーらせん( $r^2=a\theta$ )と黄金角が結びつくと、原点付近から無限遠まで**一様な点分布**が形成される。
- 無限に大きいヒマワリを仮想する。それが無限遠で一様分布になるためには**フェルマーらせん**で、 $\alpha$ の連分数展開の**部分商が有限**に限ることが証明されている。